# Package 'segmented'

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Desc	ription Fitting regression models where, in addition to possible linear terms, one or more covariates have segmented (i.e., broken-line or piece-wise linear) or stepmented (i.e. piece-wise constant) effects. Multiple breakpoints for the same variable are allowed.  The estimation method is discussed in Muggeo (2003, <doi:10.1002 sim.1545="">) and illustrated in Muggeo (2008, <https: doc="" rnews="" rnews_2008-1.pdf="" www.r-project.org="">). An approach for hypothesis testing is presented in Muggeo (2016, <doi:10.1080 00949655.2016.1149855="">), and interval estimation for the breakpoint is discussed in Muggeo (2017, <doi:10.1111 anzs.12200="">). Segmented mixed models, i.e. random effects in the change point, are discussed in Muggeo (2014, <doi:10.1177 1471082x13504721="">). Estimation of piecewise-constant relationships and changepoints (mean-shift models) is discussed in Fasola et al. (2018, <doi:10.1007 s00180-017-0740-4="">).</doi:10.1007></doi:10.1177></doi:10.1111></doi:10.1080></https:></doi:10.1002>	
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segmented-package	Segmented Relationships in Regression Models with Breakpoints / Changepoints Estimation (with Possibly Random Effects)

#### **Description**

Estimation and inference of regression models with piecewise linear relationships, also known as segmented regression models, with a number of break-points fixed or to be 'selected'. Random effects changepoints are also allowed since version 1.6-0, and since version 2.0-0 it is also possible to fit regression models with piecewise constant (or 'stepmented') relationships.

#### **Details**

Package: segmented Type: Package Version: 2.1-3 Date: 2024-10-25

License: GPL

Package segmented aims to estimate linear and generalized linear models (and virtually any regression model) having one or more segmented or stepmented relationships in the linear predictor. Estimates of the slopes and breakpoints are provided along with standard errors. The package includes testing/estimating functions and methods to print, summarize and plot the results.

The algorithms used by segmented are *not* grid-search. They are iterative procedures (Muggeo, 2003; Fasola et al., 2018) that need starting values *only* for the breakpoint parameters and therefore they are quite efficient even with several breakpoints to be estimated. Moreover since version 0.2-9.0, segmented implements the bootstrap restarting (Wood, 2001) to make the algorithms less sensitive to the starting values (which can be also omitted by the user).

Since version 0.5-0.0 a default method segmented.default has been added. It may be employed to include segmented relationships in *general* regression models where specific methods do not exist. Examples include quantile, Cox, and lme regressions where the random effects do not refer to the breakpoints; see segmented.lme to include random changepoints. segmented.default includes some examples.

Since version 1.0-0 the estimating algorithm has been slight modified and it appears to be much stabler (in examples with noisy segmented relationhips and flat log likelihoods) then previous versions.

Hypothesis testing (about the existence of the breakpoint) and confidence intervals are performed via appropriate methods and functions.

A tentative approach to deal with unknown number of breakpoints is also provided, see option fix.npsi in seg.control. Also, as version 1.3-0, the selgmented function has been introduced to select the number of breakpoints via the BIC or sequential hypothesis testing.

Since version 1.6-0, estimation of segmented mixed models has been introduced, see segmented. Ime and related function. Since version 2.0-0, it is possible to fit segmented relationships with constraints on the slopes, see segreg.

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Finally, since 2.0-0, it is possible to fit (G)LM wherein one or more covariates have a stepmented (i.e. a step-function like) relationship, see stepmented.

## Author(s)

Vito M.R. Muggeo <vito.muggeo@unipa.it>

#### References

Muggeo V.M.R., Atkins D.C., Gallop R.J., Dimidjian S. (2014) Segmented mixed models with random changepoints: a maximum likelihood approach with application to treatment for depression study. *Statistical Modelling*, **14**, 293-313.

Muggeo, V.M.R. (2017) Interval estimation for the breakpoint in segmented regression: a smoothed score-based approach. *Australian & New Zealand Journal of Statistics*, **59**, 311–322.

Fasola S, Muggeo V.M.R., Kuchenhoff, H. (2018) A heuristic, iterative algorithm for change-point detection in abrupt change models, *Computational Statistics*, **2**, 997–1015.

Muggeo, V.M.R. (2016) Testing with a nuisance parameter present only under the alternative: a score-based approach with application to segmented modelling. *J of Statistical Computation and Simulation* **86**, 3059–3067.

Davies, R.B. (1987) Hypothesis testing when a nuisance parameter is present only under the alternative. *Biometrika* **74**, 33–43.

Seber, G.A.F. and Wild, C.J. (1989) Nonlinear Regression. Wiley, New York.

Bacon D.W., Watts D.G. (1971) Estimating the transistion between two intersecting straight lines. *Biometrika* **58**: 525 – 534.

Muggeo, V.M.R. (2003) Estimating regression models with unknown break-points. *Statistics in Medicine* **22**, 3055–3071.

Muggeo, V.M.R. (2008) Segmented: an R package to fit regression models with broken-line relationships. *R News* **8/1**, 20–25.

Muggeo, V.M.R., Adelfio, G. (2011) Efficient change point detection in genomic sequences of continuous measurements. *Bioinformatics* **27**, 161–166.

Wood, S. N. (2001) Minimizing model fitting objectives that contain spurious local minima by bootstrap restarting. *Biometrics* **57**, 240–244.

Muggeo, V.M.R. (2010) Comment on 'Estimating average annual per cent change in trend analysis' by Clegg et al., Statistics in Medicine; 28, 3670-3682. *Statistics in Medicine*, **29**, 1958–1960.

aapc

Average annual per cent change in segmented trend analysis

## **Description**

Computes the average annual per cent change to summarize piecewise linear relationships in segmented regression models.

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## Usage

```
aapc(ogg, parm, exp.it = FALSE, conf.level = 0.95, wrong.se = TRUE,
    .vcov=NULL, .coef=NULL, ...)
```

## **Arguments**

ogg	the fitted model returned by segmented.
parm	the <i>single</i> segmented variable of interest. It can be missing if the model includes a single segmented covariate. If missing and ogg includes several segmented variables, the first one is considered.
exp.it	logical. If TRUE, the per cent change is computed, namely $\exp(\hat{\mu})-1$ where $\mu=\sum_j \beta_j w_j$ , see 'Details'.
conf.level	the confidence level desidered.
wrong.se	logical, if TRUE, the 'wrong" standard error (as discussed in Clegg et al. (2009)) ignoring uncertainty in the breakpoint estimate is returned as an attribute "wrong.se".
.vcov	The <i>full</i> covariance matrix of estimates. If unspecified (i.e. NULL), the covariance matrix is computed internally by $vcov(ogg,)$ .
.coef	The regression parameter estimates. If unspecified (i.e. NULL), it is computed internally by coef(ogg).
	further arguments to be passed on to vcov.segmented(), such as var.diff or is.

# **Details**

To summarize the fitted piecewise linear relationship, Clegg et al. (2009) proposed the 'average annual per cent change' (AAPC) computed as the sum of the slopes  $(\beta_j)$  weighted by corresponding covariate sub-interval width  $(w_j)$ , namely  $\mu = \sum_j \beta_j w_j$ . Since the weights are the breakpoint differences, the standard error of the AAPC should account for uncertainty in the breakpoint estimate, as discussed in Muggeo (2010) and implemented by aapc().

# Value

aapc returns a numeric vector including point estimate, standard error and confidence interval for the AAPC relevant to variable specified in parm.

## Note

exp.it=TRUE would be appropriate only if the response variable is the log of (any) counts.

## Author(s)

Vito M. R. Muggeo, <vito.muggeo@unipa.it>

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## References

Clegg LX, Hankey BF, Tiwari R, Feuer EJ, Edwards BK (2009) Estimating average annual per cent change in trend analysis. *Statistics in Medicine*, **28**; 3670-3682.

Muggeo, V.M.R. (2010) Comment on 'Estimating average annual per cent change in trend analysis' by Clegg et al., Statistics in Medicine; 28, 3670-3682. *Statistics in Medicine*, **29**, 1958–1960.

## **Examples**

```
set.seed(12)
x<-1:20
y<-2-.5*x+.7*pmax(x-9,0)-.8*pmax(x-15,0)+rnorm(20)*.3
o<-lm(y^x)
os<-segmented(o, psi=c(5,12))
aapc(os)</pre>
```

broken.line

Fitted values for segmented relationships

# **Description**

Given a segmented model (typically returned by a segmented method), broken.line computes the fitted values (and relevant standard errors) for the specified 'segmented' relationship.

# Usage

```
broken.line(ogg, term = NULL, link = TRUE, interc=TRUE, se.fit=TRUE, isV=FALSE,
   .vcov=NULL, .coef=NULL, ...)
```

ogg	A fitted object of class segmented (returned by any segmented method).
term	Three options. i) A named list (whose name should be one of the segmented covariates in the model ogg) including the covariate values for which segmented predictions should be computed; ii) a character meaning the name of any segmented covariate in the model (and predictions corresponding to the observed covariate values are returned); iii) It can be NULL if the model includes a single segmented covariate (and predictions corresponding to the observed covariate values are returned).
link	Should the predictions be computed on the scale of the link function if ogg is a segmented glm fit? Default to TRUE.
interc	Should the model intercept be added? (provided it exists).
se.fit	If TRUE also standard errors for predictions are returned.
isV	A couple of logicals indicating if the segmented terms $(x-\psi)_+$ and $I(x>\psi)$ in the model matrix should be replaced by their smoothed counterparts when computing the standard errors. If a single logical is provided, it is applied to both terms

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.VCOV	Optional. The <i>full</i> covariance matrix of estimates. If NULL (and se.fit=TRUE), the matrix is computed internally via vcov.segmented().
.coef	The regression parameter estimates. If unspecified (i.e. NULL), it is computed internally by coef(ogg).
•••	Additional arguments to be passed on to vcov.segmented() when computing the standard errors for the predictions, namely is, var.diff, p.df. See summary.segmented and vcov.segmented.

## **Details**

If term=NULL or term is a valid segmented covariate name, predictions for that segmented variable are the relevant fitted values from the model. If term is a (correctly named) list with numerical values, predictions corresponding to such specified values are computed. If link=FALSE and ogg inherits from the class "glm", predictions and possible standard errors are returned on the response scale. The standard errors come from the Delta method. Argument link is ignored whether ogg does not inherit from the class "glm".

#### Value

A list having one component if (if se.fit=FALSE), and two components (if se.fit=TRUE) list representing predictions and standard errors for the segmented covariate values.

## Note

This function was written when there was not predict. segmented (which is more general).

## Author(s)

```
Vito M. R. Muggeo
```

#### See Also

```
segmented, predict.segmented, plot.segmented, vcov.segmented
```

```
set.seed(1234)
z<-runif(100)
y<-rpois(100,exp(2+1.8*pmax(z-.6,0)))
o<-glm(y~z,family=poisson)
o.seg<-segmented(o,seg.Z=~z)
## Not run: plot(z,y)
## Not run: points(z,broken.line(o.seg,link=FALSE)$fit,col=2) #ok, but use plot.segmented()!</pre>
```

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ented Confidence intervals for breakpo	breakpoints
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# Description

Computes confidence intervals for the breakpoints in a fitted 'segmented' model.

# Usage

```
## S3 method for class 'segmented'
confint(object, parm, level=0.95, method=c("delta", "score", "gradient"),
   rev.sgn=FALSE, var.diff=FALSE, is=FALSE, digits=max(4, getOption("digits") - 1),
   .coef=NULL, .vcov=NULL, ...)
```

object	a fitted segmented object.
parm	the segmented variable of interest. If missing the first segmented variable in object is considered.
level	the confidence level required, default to 0.95.
method	which confidence interval should be computed. One of "delta", "score", or "gradient". Can be abbreviated.
rev.sgn	vector of logicals. The length should be equal to the length of parm; recycled otherwise. when TRUE it is assumed that the current parm is 'minus' the actual segmented variable, therefore the sign is reversed before printing. This is useful when a null-constraint has been set on the last slope.
var.diff	logical. If method="delta", and there is a single segmented variable, var.diff=TRUE leads to standard errors based on sandwich-type formula of the covariance matrix. See Details in summary.segmented.
is	logical. If method="delta", is=TRUE means that the full covariance matrix is computed via vcov(, is=TRUE)
digits	controls the number of digits to print when returning the output.
.coef	The regression parameter estimates. If unspecified (i.e. NULL), it is computed internally by coef(object).
.vcov	The <i>full</i> covariance matrix of estimates. If unspecified (i.e. NULL), the covariance matrix is computed internally by vcov(object).
	additional parameters referring to Score-based confidence intervals, such as "h", "d.h", "bw", "msgWarn", and "n.values" specifying the number of points used to profile the Score (or Gradient) statistic.

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#### **Details**

confint.segmented computes confidence limits for the breakpoints. Currently there are three options, see argument method. method="delta" uses the standard error coming from the Delta method for the ratio of two random variables. This value is an approximation (slightly) better than the one reported in the 'psi' component of the list returned by any segmented method. The resulting confidence intervals are based on the asymptotic Normal distribution of the breakpoint estimator which is reliable just for clear-cut kink relationships. See Details in segmented. method="score" or method="gradient" compute the confidence interval via profiling the Score or the Gradient statistics smoothed out by the induced smoothing paradigm, as discussed in the reference below.

#### Value

A matrix including point estimate and confidence limits of the breakpoint(s) for the segmented variable possibly specified in parm.

#### Note

Currently method="score" or method="gradient" only works for segmented *linear* model. For segmented *generalized linear* model, currently only method="delta" is available.

#### Author(s)

Vito M.R. Muggeo

## References

Muggeo, V.M.R. (2017) Interval estimation for the breakpoint in segmented regression: a smoothed score-based approach. *Australian & New Zealand Journal of Statistics* **59**, 311–322.

## See Also

segmented and lines. segmented to plot the estimated breakpoints with corresponding confidence intervals.

```
set.seed(10)
x<-1:100
z<-runif(100)
y<-2+1.5*pmax(x-35,0)-1.5*pmax(x-70,0)+10*pmax(z-.5,0)+rnorm(100,0,2)
out.lm<-lm(y~x)
o<-segmented(out.lm,seg.Z=~x+z,psi=list(x=c(30,60),z=.4))
confint(o) #delta CI for the 1st variable
confint(o, "x", method="score") #also method="g"</pre>
```

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confint.segmented.lme Confidence intervals in segmented mixed models

## **Description**

Computes confidence intervals for all regression parameters, including the the breakpoint, in a fitted 'segmented mixed' model.

## Usage

```
## S3 method for class 'segmented.lme'
confint(object, parm, level = 0.95, obj.boot, ...)
```

## **Arguments**

object A fit object returned by segmented.lme.

parm A vector of numbers indicating which parameters should be considered. If miss-

ing all parameters.

level The confidence level.

obj.boot The possible list including the bootstrap distributions of the regression coeffi-

cients. Such list is returned by vcov.segmented.lme(.., ret.b=TRUE)

... if obj.boot is missing and bootstrap CIs are requested, additional optional ar-

guments, such as B, seed, and it.max.b, to be used in computations of the boot

distributions.

#### **Details**

If obj.boot is provided or ... includes the argument B>0, confidence intervals are computed by exploiting the bootstrap distributions.

## Value

A matrix (or a list of matrices if bootstrap ci are requested) including the confidence intervals for the model parameters.

## Warning

All the functions for segmented mixed models (\*.segmented.lme) are still at an experimental stage

## Author(s)

Vito Muggeo

## See Also

```
vcov.segmented.lme
```

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# **Examples**

```
## Not run:
confint(os) #asymptotic CI

confint(os, B=50) #boot CIs

#it is possible to obtain the boot distribution beforehand
ob <-vcov(os, B=50, ret.b=TRUE)
confint(os, obj.boot=ob) #boot CI

## End(Not run)</pre>
```

confint.stepmented

Confidence intervals for jumpoints in stepmented regression

# Description

Computes confidence intervals for the changepoints (or jumpoints) in a fitted 'stepmented' model.

# Usage

```
## S3 method for class 'stepmented'
confint(object, parm, level=0.95, method=c("delta", "score", "gradient"),
    round=TRUE, cheb=FALSE, digits=max(4, getOption("digits") - 1),
    .coef=NULL, .vcov=NULL, ...)
```

object	a fitted stepmented object.
parm	the stepmented variable of interest. If missing the first stepmented variable in object is considered.
level	the confidence level required, default to 0.95.
method	which confidence interval should be computed. One of "delta", "score", or "gradient". Can be abbreviated. Currently only "delta" allowed.
round	logical. Should the values (estimates and lower/upper limits) rounded to the smallest observed value?
cheb	logical. If TRUE, the confidence limits are computed using the Chebyshev inequality which yields conservative confidence intervals but it is 'robust' to the non-normality of the changepoint sampling distribution.
digits	controls the number of digits to print when returning the output.
.coef	The regression parameter estimates. If unspecified (i.e. NULL), it is computed internally by coef(object).
.vcov	The <i>full</i> covariance matrix of estimates. If unspecified (i.e. NULL), the covariance matrix is computed internally by vcov(object).
	additional arguments passed to vcov.stepmented, namely k.

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#### **Details**

confint.stepmented computes confidence limits for the changepoints. Currently the only option is 'delta', i.e. to compute the approximate covariance matrix via a smoothing approximation (see vcov.stepmented) and to build the limits using the standard Normal quantiles. Note that, the limits are rounded to the lowest observed value, thus the resulting confidence interval might not be symmetric if the stepmented covariate has not equispaced values.

#### Value

A matrix including point estimate and confidence limits of the breakpoint(s) for the stepmented variable possibly specified in parm.

#### Note

Currently only method='delta' is allowed.

## Author(s)

Vito M.R. Muggeo

#### See Also

stepmented and lines.segmented to plot the estimated breakpoints with corresponding confidence intervals.

# **Examples**

```
set.seed(10)
x<-1:100
z<-runif(100)
y<-2+2.5*(x>45)-1.5*(x>70)+z+rnorm(100)
o<-stepmented(y, npsi=2)

confint(o) #round=TRUE is default
confint(o, round=FALSE)</pre>
```

davies.test

Testing for a change in the slope

# **Description**

Given a generalized linear model, the Davies' test can be employed to test for a non-constant regression parameter in the linear predictor.

#### Usage

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## Arguments

obj a fitted model typically returned by glm or lm. Even an object returned by

segmented can be set (e.g. if interest lies in testing for an additional break-

point).

seg.Z a formula with no response variable, such as seg.Z=~x1, indicating the (con-

tinuous) segmented variable being tested. Only a single variable may be tested and an error is printed when seg.Z includes two or more terms. seg.Z can be omitted if i)obj is a segmented fit with a single segmented covariate (and that variable is taken), or ii)if it is a "lm" or "glm" fit with a single covariate (and that

variable is taken)

k number of points where the test should be evaluated. See Details.

alternative a character string specifying the alternative hypothesis (relevant to the slope

difference parameter).

type the test statistic to be used (only for GLM, default to lrt). Ignored if obj is a

simple linear model.

values optional. The evaluation points where the Davies approximation is computed.

See Details for default values.

dispersion the dispersion parameter for the family to be used to compute the test statistic.

When NULL (the default), it is inferred from obj. Namely it is taken as 1 for the Binomial and Poisson families, and otherwise estimated by the residual Chisquared statistic (calculated from cases with non-zero weights) divided by the

residual degrees of freedom.

## **Details**

davies. test tests for a non-zero difference-in-slope parameter of a segmented relationship. Namely, the null hypothesis is  $H_0: \beta = 0$ , where  $\beta$  is the difference-in-slopes, i.e. the coefficient of the segmented function  $\beta(x-\psi)_+$ . The hypothesis of interest  $\beta=0$  means no breakpoint. Roughtly speaking, the procedure computes k 'naive' (i.e. assuming fixed and known the breakpoint) test statistics for the difference-in-slope, seeks the 'best' value and corresponding naive p-value (according to the alternative hypothesis), and then corrects the selected (minimum) p-value by means of the k values of the test statistic. If obj is a LM, the Davies (2002) test is implemented. This approach works even for small samples. If obj represents a GLM fit, relevant methods are described in Davies (1987), and the Wald or the Likelihood ratio test statistics can be used, see argument type. This is an asymptotic test. The k evaluation points are k equally spaced values between the second and the second-last values of the variable reported in seg. Z. k should not be small; I find no important difference for k larger than 10, so default is k=10.

## Value

A list with class 'htest' containing the following components:

method title (character)

data.name the regression model and the segmented variable being tested

statistic the point within the range of the covariate in seg. Z at which the maximum (or

the minimum if alternative="less") occurs

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parameter number of evaluation points

p.value the adjusted p-value

process a two-column matrix including the evaluation points and corresponding values

of the test statistic

## Warning

The Davies test is *not* aimed at obtaining the estimate of the breakpoint. The Davies test is based on k evaluation points, thus the value returned in the statistic component (and printed as "'best' at") is the best among the k points, and typically it will differ from the maximum likelihood estimate returned by segmented. Use segmented if you are interested in the point estimate.

To test for a breakpoint in *linear* models with small samples, it is suggested to use davies.test() with objects of class "lm". If obj is a "glm" object with gaussian family, davies.test() will use an approximate test resulting in smaller p-values when the sample is small. However if the sample size is large (n>300), the exact Davies (2002) upper bound cannot be computed (as it relies on gamma() function) and the *approximate* upper bound of Davies (1987) is returned.

#### Note

Strictly speaking, the Davies test is not confined to the segmented regression; the procedure can be applied when a nuisance parameter vanishes under the null hypothesis. The test is slightly conservative, as the computed p-value is actually an upper bound.

Results should change slightly with respect to previous versions where the evaluation points were computed as k equally spaced values between the second and the second last observed values of the segmented variable.

# Author(s)

Vito M.R. Muggeo

## References

Davies, R.B. (1987) Hypothesis testing when a nuisance parameter is present only under the alternative. *Biometrika* **74**, 33–43.

Davies, R.B. (2002) Hypothesis testing when a nuisance parameter is present only under the alternative: linear model case. *Biometrika* **89**, 484–489.

## See Also

See also pscore.test which is more powerful, especially when the signal-to-noise ratio is low.

```
## Not run:
set.seed(20)
z<-runif(100)
x<-rnorm(100,2)
y<-2+10*pmax(z-.5,0)+rnorm(100,0,3)</pre>
```

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```
o<-lm(y~z+x)
davies.test(o,~z)
davies.test(o,~x)

o<-glm(y~z+x)
davies.test(o,~z) #it works but the p-value is too small..
## End(Not run)</pre>
```

down

Down syndrome in babies

# Description

The down data frame has 30 rows and 3 columns. Variable cases means the number of babies with Down syndrome out of total number of births for mothers with mean age age.

# Usage

```
data(down)
```

## **Format**

A data frame with 30 observations on the following 3 variables.

age the mothers' mean age.

births count of total births.

cases count of babies with Down syndrome.

#### Source

Davison, A.C. and Hinkley, D. V. (1997) *Bootstrap Methods and their Application*. Cambridge University Press.

## References

Geyer, C. J. (1991) Constrained maximum likelihood exemplified by isotonic convex logistic regression. *Journal of the American Statistical Association* **86**, 717–724.

```
data(down)
```

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draw.history

History for the breakpoint estimates

# Description

Displays breakpoint iteration values for segmented fits.

## Usage

```
draw.history(obj, term, ...)
```

## **Arguments**

obj a segmented fit returned by any "segmented" method.

term a character to mean the 'segmented' variable whose breakpoint values through-

out iterations have to be displayed.

... graphic parameters to be passed to matplot().

## **Details**

For a given term in a segmented fit, draw.history() produces two plots. On the left panel it displays the different breakpoint values obtained during the estimating process, since the starting values up to the final ones, while on the right panel the objective values at different iterations. When bootstrap restarting is employed, draw.history() produces two plots, the values of objective function and the number of distinct solutions against the bootstrap replicates.

## Value

None.

# Author(s)

Vito M.R. Muggeo

```
data(stagnant)
os<-segmented(lm(y~x,data=stagnant),seg.Z=~x,psi=-.8)
# draw.history(os) #diagnostics with boot restarting
os<-segmented(lm(y~x,data=stagnant),seg.Z=~x,psi=-.8, control=seg.control(n.boot=0))
# draw.history(os) #diagnostics without boot restarting</pre>
```

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fitted.segmented.lme Fitted values for segmented mixed fits

# Description

Computes fitted values at different levels of nesting for segmented mixed objects

# Usage

```
## S3 method for class 'segmented.lme'
fitted(object, level = 1, sort=TRUE, ...)
```

# Arguments

object Object of class "segmented.lme"

the level to be considered. Currently only levels 0 or 1 are allowed.

Sort If TRUE, the fitted values are sorted by the names of the 'id' levels.

... Ignored

# **Details**

Currently it works only if level=1

#### Value

A numeric object including the fitted values at the specified level of nesting.

# Warning

All the functions for segmented mixed models (\*.segmented.lme) are still at an experimental stage

# Author(s)

Vito Muggeo

## See Also

```
summary.segmented.lme
```

18 globTempAnom

globTempAnom

Global temperature anomalies 1850-2023

# **Description**

The globTempAnom data frame includes the global surface temperature anomalies from 1850 to 2023

# Usage

data(globTempAnom)

#### **Format**

The included variables are (clearly).

Year the calendar year.

Anomaly the temperature anomalies computed as differences of the annual (average) measurement with respect to the 20th century average (1901-2000).

# **Details**

Data refer to averages measurements referring to land and ocean surface of Northern and Southern hemisphere.

# Source

https://www.ncei.noaa.gov/access/monitoring/global-temperature-anomalies/anomalies

## References

There are several references using such dataset, e.g.

Cahill, N., Rahmstorf, S., and Parnell, A. C. (2015). Change points of global temperature. *Environmental Research Letters*, 10: 1-6.

# **Examples**

data(globTempAnom)

intercept 19

intercept	Intercept estimates from segmented relationships

# Description

Computes the intercepts of each 'segmented' relationship in the fitted model.

# Usage

```
intercept(ogg, parm, rev.sgn = FALSE, var.diff=FALSE,
    .vcov=NULL, .coef=NULL, digits = max(4, getOption("digits") - 2),...)
```

# **Arguments**

ogg	an object of class "segmented", returned by any segmented method.
parm	the segmented variable whose intercepts have to be computed. If missing all the segmented variables in the model are considered.
rev.sgn	vector of logicals. The length should be equal to the length of parm, but it is recycled otherwise. When TRUE it is assumed that the current parm is 'minus' the actual segmented variable, therefore the order is reversed before printing. This is useful when a null-constraint has been set on the last slope.
var.diff	Currently ignored as only point estimates are computed.
.vcov	The <i>full</i> covariance matrix of estimates. If unspecified (i.e. NULL), the covariance matrix is computed internally by vcov(ogg).
.coef	The regression parameter estimates. If unspecified (i.e. NULL), it is computed internally by coef(ogg).
digits	controls number of digits in the returned output.
	Further arguments to be passed on to vcov.segmented, such as var.diff and is. See Details in vcov.segmented.

## **Details**

A broken-line relationship means that a regression equation exists in the intervals 'min(x) to  $\psi_1$ ', ' $\psi_1$  to  $\psi_2$ ', and so on. intercept computes point estimates of the intercepts of the different regression equations for each segmented relationship in the fitted model.

## Value

intercept returns a list of one-column matrices. Each matrix represents a segmented relationship.

# Author(s)

Vito M. R. Muggeo, <vito.muggeo@unipa.it>

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# See Also

See also slope to compute the slopes of the different regression equations for each segmented relationship in the fitted model.

# **Examples**

```
## see ?slope
## Not run:
intercept(out.seg)
## End(Not run)
```

lines.segmented

Bars for interval estimate of the breakpoints

# **Description**

Draws bars relevant to breakpoint estimates (point estimate and confidence limits) on the current device

# Usage

```
## S3 method for class 'segmented'
lines(x, term, bottom = TRUE, shift=FALSE, conf.level = 0.95, k = 50,
    pch = 18, rev.sgn = FALSE, .vcov=NULL, .coef=NULL, ...)
```

x	an object of class segmented.
term	the segmented variable of the breakpoints being drawn. It may be unspecified when there is a single segmented variable.
bottom	logical, indicating if the bars should be plotted at the bottom (TRUE) or at the top (FALSE).
shift	logical, indicating if the bars should be 'shifted' on the y-axis before plotting. Useful for multiple breakpoints with overlapped confidence intervals.
conf.level	the confidence level of the confidence intervals for the breakpoints.
k	a positive integer regulating the vertical position of the drawn bars. See Details.
pch	either an integer specifying a symbol or a single character to be used in plotting the point estimates of the breakpoints. See points.
rev.sgn	should the signs of the breakpoint estimates be changed before plotting? see Details.
.vcov	The <i>full</i> covariance matrix of estimates. If unspecified (i.e. NULL), the covariance matrix is computed internally by vcov(x).
.coef	The regression parameter estimates. If unspecified (i.e. $NULL$ ), it is computed internally by $coef(x)$ .
	further arguments passed to arrows, for instance 'col' that can be a vector.

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## **Details**

lines.segmented simply draws on the current device the point estimates and relevant confidence limits of the estimated breakpoints from a "segmented" object. The y coordinate where the bars are drawn is computed as usr[3]+h if bottom=TRUE or usr[4]-h when bottom=FALSE, where h=(usr[4]-usr[3])/abs(k) and usr are the extremes of the user coordinates of the plotting region. Therefore for larger values of k the bars are plotted on the edges. The argument rev.sgn allows to change the sign of the breakpoints before plotting. This may be useful when a null-right-slope constraint is set.

#### See Also

plot. segmented to plot the fitted segmented lines, and points. segmented to add the fitted join-points.

## **Examples**

```
## See ?plot.segmented
```

lines.stepmented

Bars for interval estimate of the breakpoints

# Description

Draws bars relevant to breakpoint estimates (point estimate and confidence limits) on the current device

## Usage

```
## S3 method for class 'stepmented'
lines(x, term, bottom = TRUE, shift=FALSE, conf.level = 0.95, k = 50,
    pch = 18, .vcov=NULL, .coef=NULL, ...)
```

X	an object of class stepmented.
term	the stepmented variable of the breakpoints being drawn. It may be unspecified when there is a single stepmented variable.
bottom	logical, indicating if the bars should be plotted at the bottom (TRUE) or at the top (FALSE).
shift	logical, indicating if the bars should be 'shifted' on the y-axis before plotting. Useful for multiple breakpoints with overlapped confidence intervals.
conf.level	the confidence level of the confidence intervals for the breakpoints.
k	a positive integer regulating the vertical position of the drawn bars. See Details.
pch	either an integer specifying a symbol or a single character to be used in plotting the point estimates of the breakpoints. See points.

.VCOV	The <i>full</i> covariance matrix of estimates. If unspecified (i.e. NULL), the covariance matrix is computed internally by $vcov(x)$ .
.coef	The regression parameter estimates. If unspecified (i.e. $NULL$ ), it is computed internally by $coef(x)$ .
	further arguments passed to arrows, for instance 'col' that can be a vector.

## **Details**

lines.stepmented simply draws on the current device the point estimates and relevant confidence limits of the estimated breakpoints from a "stepmented" object. The y coordinates where the bars are drawn is computed as usr[3]+h if bottom=TRUE or usr[4]-h when bottom=FALSE, where h=(usr[4]-usr[3])/abs(k) and usr are the extremes of the user coordinates of the plotting region. Therefore for larger values of k the bars are plotted on the edges.

## See Also

```
plot.stepmented to plot the fitted stepmented lines
```

# **Examples**

```
## See ?plot.stepmented
```

```
model.matrix.segmented
```

Design matrix for segmented fits

# Description

This function builds the model matrix for segmented fits.

## Usage

```
## S3 method for class 'segmented'
model.matrix(object, ...)
```

# **Arguments**

```
object A segmented fit
... additional arguments
```

## **Details**

```
model.matrix.segmented
```

## Value

The design matrix for a segmented regression model with the specified formula and data

## Author(s)

Vito Muggeo

#### See Also

See Also as model.matrix

model.matrix.stepmented

Design matrix for stepmented fits

# **Description**

This function builds the model matrix for stepmented fits.

## Usage

```
## S3 method for class 'stepmented'
model.matrix(object, type=c("cdf","abs","none"), k=NULL, ...)
```

# **Arguments**

object	A stepmented fit
k	The (negative) exponent of the sample size to approximate the absolute value; see vcov.stepmented for details.
type	The approximation for the indicator function/absolute value. If "none", the simple matrix with the original indicator values is returned. type='abs' is not yet allowed.
	additional arguments

## **Details**

If type="none", model.matrix.stepmented return the design matrix including the indicator function values and ignoring the psi terms.

## Value

The design matrix for a stepmented regression model with the specified formula and data

# Author(s)

Vito Muggeo

#### See Also

See Also as model.matrix, vcov.stepmented

24 plant

plant

Plan organ dataset

## Description

The plant data frame has 103 rows and 3 columns.

## Usage

```
data(plant)
```

#### **Format**

A data frame with 103 observations on the following 3 variables:

```
y measurements of the plant organ.
```

time times where measurements took place.

group three attributes of the plant organ, RKV, RKW, RWC.

## **Details**

Three attributes of a plant organ measured over time where biological reasoning indicates likelihood of multiple breakpoints. The data are scaled to the maximum value for each attribute and all attributes are measured at each time.

## **Source**

The data have been kindly provided by Dr Zongjian Yang at School of Land, Crop and Food Sciences, The University of Queensland, Brisbane, Australia.

```
## Not run:
data(plant)

lattice::xyplot(y~time,groups=group,auto.key=list(space="right"), data=plant)

o<-segreg(y~ 0+group+seg(time, by=group, npsi=2), data=plant)
summary(o)

par(mfrow=c(1,2))
plot(y~time, data=plant)
plot(o, term=1:3, add=TRUE, leg=NA, psi.lines=TRUE) #add the lines to the current plot
plot(o, term=1:3, col=3:5, res.col=3:5, res=TRUE, leg="bottomright")

## End(Not run)</pre>
```

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plot.segmented	Plot method for segmented objects

# Description

Takes a fitted segmented object returned by segmented() and plots (or adds) the fitted broken-line relationship for the selected segmented term.

# Usage

```
## S3 method for class 'segmented'
plot(x, term, add=FALSE, res=FALSE, conf.level=0, interc=TRUE, link=TRUE,
    res.col=grey(.15, alpha = .4), rev.sgn=FALSE, const=NULL, shade=FALSE, rug=!add,
    dens.rug=FALSE, dens.col = grey(0.8), transf=I, isV=FALSE, is=FALSE, var.diff=FALSE,
    p.df="p", .vcov=NULL, .coef=NULL, prev.trend=FALSE, smoos=NULL, hide.zeros=FALSE,
    leg="topleft", psi.lines=FALSE, ...)
```

X	a fitted segmented object.
term	Numerical or character to indicate the segmented variable having the piece-wise relationship to be plotted. If there is a single segmented variable in the fitted model x, term can be omitted. If vector, multiple segmented lines will be drawn on the same plot.
add	when TRUE the fitted lines are added to the current device.
res	when TRUE the fitted lines are plotted along with corresponding partial residuals. See Details.
conf.level	If greater than zero, it means the confidence level at which the pointwise confidence itervals have to be plotted.
interc	If TRUE the computed segmented components include the model intercept (if it exists).
link	when TRUE (default), the fitted lines are plotted on the link scale, otherwise they are tranformed on the response scale before plotting. Ignored for linear segmented fits.
res.col	when res=TRUE it means the color of the points representing the partial residuals.
rev.sgn	when TRUE it is assumed that current term is 'minus' the actual segmented variable, therefore the sign is reversed before plotting. This is useful when a null-constraint has been set on the last slope.
const	constant to add to each fitted segmented relationship (on the scale of the linear predictor) before plotting. If const=NULL and the fit includes a segmented interaction term (obtained via seg(,by) in the formula), the group-specific intercept is included.

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shade	if TRUE and conf.level>0 it produces shaded regions (in grey color) for the pointwise confidence intervals embracing the fitted segmented line.
rug	when TRUE the covariate values are displayed as a rug plot at the foot of the plot. Default is to ! add.
dens.rug	when TRUE then smooth covariate distribution is plotted on the x-axis.
dens.col	if dens.rug=TRUE, it means the colour to be used to plot the density.
transf	A possible function to convert the fitted values before plotting. It is only effective if res=FALSE. If res=TRUE any transformation is ignored.
isV	logical value (to be passed to broken.line). Ignored if conf.level=0
is	logical value (to be passed to broken.line) indicating if the covariance matrix based on the induced smoothing should be used. Ignored if conf.level=0
var.diff	logical value to be passed to summary.segmented to compute dthe standard errors of fitted values (if $conf.level>0$ ).
p.df	degrees of freedom when var.diff=TRUE, see summary.segmented
.vcov	The $full$ covariance matrix of estimates to be used when conf.level>0. If unspecified (i.e. NULL), the covariance matrix is computed internally by the function vcov.segmented.
.coef	The regression parameter estimates. If unspecified (i.e. NULL), it is computed internally by $coef()$ .
prev.trend	logical. If TRUE dashed lines corresponding to the 'previous' trends (i.e. the trends if the breakpoints would not have occurred) are also drawn.
smoos	logical, indicating if the residuals (provided that res=TRUE) will be drawn using a <i>smoothed</i> scatterplot. If NULL (default) the smoothed scatterplot will be employed when the number of observation is larger than 10000.
hide.zeros	logical, indicating if the residuals (provided that res=TRUE) corresponding to the covariate zero values should be deleted. Useful when the fit includes an interaction term in the formula, such as seg(, by=), and the zeros in covariates indicate units in other groups.
leg	If the plot refers to segmented relationships in groups, i.e. term has been specified as a vector, a legend is placed at the specified leg position. Put NA not to draw the legend.
psi.lines	if TRUE vertical lines corresponding to the estimated breakpoints are also drawn. Ignored if term is not a vector.
	other graphics parameters to pass to plotting commands: 'col', 'lwd' and 'lty' (that can be vectors and are recycled if necessary, see the example below) for the fitted piecewise lines; 'ylab', 'xlab', 'main', 'sub', 'cex.axis', 'cex.lab', 'xlim' and 'ylim' when a new plot is produced (i.e. when add=FALSE); 'pch' and 'cex' for the partial residuals (when res=TRUE, res.col is for the color); col.shade for the shaded regions (provided that shade=TRUE and conf.level>0).

# **Details**

Produces (or adds to the current device) the fitted segmented relationship between the response and the selected term. If the fitted model includes just a single 'segmented' variable, term may be

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omitted.

The partial residuals are computed as 'fitted + residuals', where 'fitted' are the fitted values of the segmented relationship relevant to the covariate specified in term. Notice that for GLMs the residuals are the response residuals if link=FALSE and the working residuals if link=TRUE.

#### Value

None.

#### Note

For models with offset, partial residuals on the response scale are not defined. Thus plot.segmented does not work when link=FALSE, res=TRUE, and the fitted model includes an offset. When term is a vector and multiple segmented relationships are being drawn on the same plot, col and res.col can be vectors. Also pch, cex, lty, and lwd can be vectors, if specified.

#### Author(s)

Vito M. R. Muggeo

#### See Also

segmented to fit the model, lines.segmented to add the estimated breakpoints on the current plot. points.segmented to add the joinpoints of the segmented relationship. predict.segmented to compute standard errors and confidence intervals for predictions from a "segmented" fit.

```
set.seed(1234)
z<-runif(100)
y < -rpois(100, exp(2+1.8*pmax(z-.6,0)))
o<-glm(y~z,family=poisson)
o.seg<-segmented(o) #single segmented covariate and one breakpoint: 'seg.Z' and 'psi' can be omitted
par(mfrow=c(1,2))
plot(o.seg, conf.level=0.95, shade=TRUE)
points(o.seg, link=TRUE, col=2)
## new plot
plot(z,y)
## add the fitted lines using different colors and styles..
plot(o.seg,add=TRUE,link=FALSE,lwd=2,col=2:3, lty=c(1,3))
lines(o.seg,col=2,pch=19,bottom=FALSE,lwd=2) #for the CI for the breakpoint
points(o.seg,col=4, link=FALSE)
## using the options 'is', 'isV', 'shade' and 'col.shade'.
par(mfrow=c(1,2))
plot(o.seg, conf.level=.9, is=TRUE, isV=TRUE, col=1, shade = TRUE, col.shade=2)
plot(o.seg, conf.level=.9, is=TRUE, isV=FALSE, col=2, shade = TRUE, res=TRUE, res.col=4, pch=3)
```

28 plot.segmented.lme

plot.segmented.lme Plot method for segmented mixed objects

# Description

Takes a fitted segmented.lme object returned by segmented.lme() and plots (or adds) the fitted broken-line relationship for the segmented term.

## Usage

```
## S3 method for class 'segmented.lme'
plot(x, level=1, id = NULL, res = TRUE, pop = FALSE, yscale = 1, xscale = 1,
    n.plot, pos.leg = "topright", vline = FALSE, lines = TRUE,
    by=NULL, add=FALSE, conf.level=0, withI=TRUE, vcov.=NULL, shade=FALSE,
    drop.var=NULL, text.leg=NULL, id.name=TRUE, ...)
```

х	Object of class "segmented.lme"
level	An integer giving the level of grouping to be used when computing the segmented relationship(s). level=0 means depending on fixed effects estimates only (such estimates are also said, to some extend, 'population' or 'marginal' estimates), otherwise the segmented lines will also depend on the random effects predictions.
id	A scalar or vector meaning which subjects profiles have to be drawn. If NULL (default) all profiles are drawn. Ignored if level=0.
res	If TRUE, the data points are also drawn. Ignored if level=0.
pop	if TRUE, the fitted segmented relationships based on fixed-effects only is also portrayed. Ignored if level=0.
yscale	If $>= 0$ , the same and common y-scale is used for all 'subjects' (panels); otherwise the y-scale will depend on the actual (observed and fitted) values for each 'subject'.
xscale	If $>= 0$ , the same and common x-scale is used for all 'subjects' (panels); otherwise the x-scale will depend on the actual observed values of the segmented covariate for each 'subject'.
n.plot	a vector to be passed to par(mfrow=c()) for plotting multiple panels (should be coherent with length(id)). If missing, it is computed automatically depending on length(id). Type n.plot=1 to draw all the segmented profiles on the same panel.
pos.leg	a character ('topright', 'topleft',) meaning the location of the legend. Set $NULL$ for no legend.
vline	logical, if TRUE a vertical dashed segment is added to emphasize the breakpoint location.

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lines	logical, if FALSE points, rather than lines, are used to portray the segmented relationships.
by	A named list indicating covariate names and corresponding values affecting the fitted segmented relationship. For instance: by=list(sex="male", z=.2), provided that the variables sex and z affect the segmented relationship. Effective only if level=0.
add	If TRUE the (fixed-effect) fitted segmented relationship is added to the current plot. Effective only if level=0.
conf.level	The confidence level for pointwise confidence intervals. Effective only if level=0.
withI	If TRUE, the level 0 segmented relationship is computed with the model intercept. Effective only if level=0.
vcov.	The fixed effects covariance matrix. If NULL, it is computed by vcov.segmented.lme(). Effective only if level=0.
shade	If TRUE (and conf.level>0) the area within the pointiwise CIs is shaded. Effective only if level=0.
drop.var	Possible coefficient names to be removed before computing the segmented relationship (E.g. the group-specific intercept.).
text.leg	If specified (and pos.leg has been also specified), it is the legend text. Effective only if level=0.
id.name	If pos.leg is different from NULL, id.name=TRUE will portray the cluster variable name along the value. Namely id.name=TRUE leads to 'country = italy' on each panel, while id.name=FALSE to 'italy'.
	additional arguments, such as ylab,xlab, ylim and xlim; l.col,l.lwd,l.lty (for the fitted individual lines - can be vectors and will be recycled); p.col, p.lwd, p.lty for the population line (if pop=TRUE); col, cex, pch for the data points (if res=TRUE); t.col for the legend color, if pos.leg is not NULL. If level=0 and conf.level>0, lty and lwd can be vectors.

#### **Details**

The function plots the 'subject'-specific segmented profiles for the 'subjects' specificed in id or, if level=0, the fitted segmented relationship based on fixed effects only. The number of panels to drawn is actually the minimum between length(id) and prod(n.plot), but if n.plot=c(1,1) (or also simply n.plot=1), the 'individual' profiles will be pictured on the same panel.

# Value

A single or multiple (depending on level and id) plot showing the fitted segmented profiles.

# Warning

All the functions for segmented mixed models (\*.segmented.lme) are still at an experimental stage

## Note

If by is specified (and level=0 is set), a legend is also added in the plot reporting covariate(s) name and value affecting the segmented relationship. Set pos.leg=TRUE to have no legend. On the other hand, use text.leg to add legend reporting the covariate baseline values.

30 plot.stepmented

## Author(s)

Vito Muggeo

#### See Also

```
segmented.lme
```

# **Examples**

```
## Not run:
#continues example from segmented.lme
plot(os, yscale=-1) #different y-scales
plot(os2, n.plot=1, 1.col=2:6, 1.lwd=2) #all segmented profiles on the same plot
## End(Not run)
```

plot.stepmented

Plot method for stepmented objects

# **Description**

Takes a fitted stepmented object returned by stepmented() and plots (or adds) the fitted piecewise constant lines for the selected stepmented term.

# Usage

X	a fitted stepmented object.
term	the stepmented variable having the piece-wise constant relationship to be plotted. If there is a single stepmented variable in the fitted model x, term can be omitted.
add	when TRUE the fitted lines are added to the current device.
res	when TRUE the fitted lines are plotted along with corresponding partial residuals.
conf.level	the confidence level for the pointwise confidence intervals for the expected values.
interc	if TRUE the computed components include the model intercept (if it exists).

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add.fx	logical. If TRUE and the object fit also includes an additional term for the same stepmented variable, the plot also portrays such 'additional' term.
psi.lines	if TRUE vertical lines corresponding to the estimated changepoints are also drawn
link	if FALSE the fitted lines (and possibily the residuals) are reported on the response scale. Ignored if the fit object x is not a glm-like fit.
const	constant to add to each fitted segmented relationship (on the scale of the linear predictor) before plotting. If const=NULL and the fit includes a segmented interaction term (obtained via seg(,by) in the formula), the group-specific intercept is included.
res.col	when res=TRUE it means the color of the points representing the partial residuals.
surf	if the object fit x includes 2 stepmented covariates (x1 and x2, say) with relevant estimated breakpoints, surf=TRUE will draw on the plane x1-x2 the areas splitted according to the estimated breakpoints with corresponding estimated means superimposed.
zero.cor	see zero.cor in vcov.stepmented; effective only if conf.level>0.
heurs	logical; if TRUE, heuristic (usually somewhat conservative) confidence intervals are computed and plotted; effective only if conf.level>0.
shade	if TRUE the pointwise confidence intervals are portrayed via shaded area; effective only if conf.level>0.
se.type	which standard errors should be computed? see type in vcov.stepmented; effective only if conf.level>0.
k	The value to be passed to vcov.stepmented to computed the standard errors.
.vcov	The estimate var-covariance matrix; if NULL, it is computed internally by $\verb"vcov.step"$ mented.
leg	If the plot refers to stepmented relationships in groups, i.e. term has been specified as a vector, a legend is placed at the specified leg position. Put NA not to draw the legend.
	other graphics parameters to pass to plotting commands: 'col', 'lwd' and 'lty' (that can be vectors and are recycled if necessary, see the example below) for the fitted piecewise constant lines; 'ylab', 'xlab', 'main', 'sub', 'cex.axis', 'cex.lab', 'xlim' and 'ylim' when a new plot is produced (i.e. when add=FALSE); 'pch' and 'cex' for the partial residuals (when res=TRUE, res.col is for the color).

# **Details**

Produces (or adds to the current device) the fitted step-function like relationship between the response and the selected term. If the fitted model includes just a single 'stepmented' variable, term may be omitted. If surf=TRUE, and res=TRUE the point widths are proportional to the partial residual values.

# Value

None.

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## Note

Implementation of confidence intervals for the conditional means in stepmented regression is under development; conf.level>0 should be used with care, especially with multiple jumpoints.

## Author(s)

```
Vito M. R. Muggeo
```

## See Also

See Also as stepmented

## **Examples**

```
#Following code in stepmented..
## Not run:

par(mfrow=c(1,3))

plot(os,"x")
plot(os,"z")
plot(os,"z", add.fx=TRUE, psi.lines=FALSE )
lines(os, "z")

#display the 'surface'
par(mfrow=c(1,3))
plot(os, surf=TRUE, col=1, res.col=2)
plot(os, surf=TRUE, lty=2)
plot(x,z)
plot(os, surf=TRUE, add=TRUE, col=4, res=FALSE)

## End(Not run)
```

points.segmented

Points method for segmented objects

## **Description**

Takes a fitted segmented object returned by segmented() and adds on the current plot the join-points of the fitted broken-line relationships.

# Usage

points.segmented 33

# Arguments

X	an object of class segmented.
term	the segmented variable of interest. It may be unspecified when there is a single segmented variable.
interc	If TRUE the computed joinpoints include the model intercept (if it exists).
link	when TRUE (default), the fitted joinpoints are plotted on the link scale
rev.sgn	when TRUE, the fitted joinpoints are plotted on the 'minus' scale of the current term variable. This is useful when a null-constraint has been set on the last slope.
transf	A possible function to convert the fitted values before plotting.
. vcov	The $full$ covariance matrix of estimates. If unspecified (i.e. NULL), the covariance matrix is computed internally by vcov().
.coef	The regression parameter estimates. If unspecified (i.e. NULL), it is computed internally by $coef(x)$ .
const	A constant to be added (on the y-scale) to the values before transforming and plotting.
V	logical. If TRUE, vertical lines at the breakpoints are also drawn.
• • •	other graphics parameters to pass on to points() and segments() (if v=TRUE).

## **Details**

We call 'joinpoint' the plane point having as coordinates the breakpoints (on the x scale) and the fitted values of the segmented relationship at that breakpoints (on the y scale). points.segmented() simply adds the fitted joinpoints on the current plot. This could be useful to emphasize the changes of the piecewise linear relationship.

# See Also

plot.segmented to plot the fitted segmented lines.

```
## Not run:
#see examples in ?plot.segmented
## End(Not run)
```

34 predict.segmented

1	
predict.	segmented

Predict method for segmented model fits

# **Description**

Returns predictions and optionally associated quantities (standard errors or confidence intervals) from a fitted segmented model object.

## Usage

## **Arguments**

object	a fitted segmented model coming from segmented.(g)lm or segreg.
newdata	An optional data frame in which to look for variables with which to predict. If omitted, the fitted values are used.
se.fit	Logical. Should the standard errors be returned?
interval	Which interval? See predict.1m
type	Predictions on the link or response scale? Only if object is a segmented glm.
na.action	How to deal with missing data, if newdata include them.
level	The confidence level.
.coef	The regression parameter estimates. If unspecified (i.e. NULL), it is computed internally by coef().

# **Details**

Basically predict.segmented builds the right design matrix accounting for breakpoint and passes it to predict.lm or predict.glm depending on the actual model fit object.

further arguments.

# Value

predict.segmented produces a vector of predictions with possibly associated standard errors or confidence intervals. See predict.lm or predict.glm.

## Warning

For segmented glm fits with offset, predict.segmented returns the fitted values *including* the offset.

## Author(s)

Vito Muggeo

predict.stepmented 35

## See Also

```
segreg, segmented, plot.segmented, broken.line, predict.lm, predict.glm
```

## **Examples**

```
n=10 x=seq(-3,3,1=n) set.seed(1515) y <- (x<0)*x/2 + 1 + rnorm(x,sd=0.15) segm <- segmented(lm(y ~ x), ~ x, psi=0.5) predict(segm,se.fit = TRUE)$se.fit \#wrong (smaller) st.errors (assuming known the breakpoint) olm<-lm(y~x+pmax(x-segm$psi[,2],0)) predict(olm,se.fit = TRUE)$se.fit
```

predict.stepmented

Predict method for stepmented model fits

## **Description**

Returns predictions and optionally associated quantities (standard errors or confidence intervals) from a fitted stepmented model object.

# Usage

object	a fitted stepmented model coming from stepmented.lm or stepmented.glm.
newdata	An optional data frame in which to look for variables with which to predict. If omitted, the fitted values are used.
se.fit	Logical. Should the standard errors be returned?
interval	Which interval? See predict.lm
type	Predictions on the link or response scale? Only if object is a stepmented glm.
na.action	How to deal with missing data, if newdata include them.
level	The confidence level.
.coef	The regression parameter estimates. If unspecified (i.e. NULL), it is computed internally by coef().
. VCOV	The estimate covariance matrix. If unspecified (i.e. NULL), it is computed internally by vcov.stepmented().

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apprx.fit	The approximation of the $(x>\hat{\psi})$ used to compute the predictions/fitted values of the piece-wise relationships.
apprx.se	The same abovementioned approximation to compute the standard error.
	further arguments, for instance k to be passed to vcov.stepmented.

# **Details**

Basically predict.stepmented builds the right design matrix accounting for breakpoint and passes it to predict.lm or predict.glm depending on the actual model fit object.

## Value

predict.stepmented produces a vector of predictions with possibly associated standard errors or confidence intervals. See predict.lm, predict.glm, or predict.segmented.

# Warning

For stepmented glm fits with offset, predict.stepmented returns the fitted values *including* the offset.

## Author(s)

Vito Muggeo

# See Also

```
stepreg, stepmented, plot.stepmented, predict.lm, predict.glm
```

# **Examples**

```
n=10 x=seq(-3,3,l=n) set.seed(1515) y <- (x<0)*x/2 + 1 + rnorm(x,sd=0.15) segm <- segmented(lm(y <math>\sim x), \sim x, psi=0.5) predict(segm,se.fit = TRUE)$se.fit
```

print.segmented

Print method for the segmented class

# **Description**

Printing the most important features and coefficients (including the breakpoints) of a segmented model.

print.segmented.lme 37

## Usage

```
## S3 method for class 'segmented'
print(x, digits = max(3, getOption("digits") - 3), ...)
## S3 method for class 'segmented'
coef(object, include.psi=FALSE, ...)
```

## **Arguments**

X	object of class segmented
digits	number of digits to be printed
object	object of class segmented
include.psi	logical. If TRUE, the breakpoints are returned along with the regression coefficients $% \left( 1\right) =\left( 1\right) \left( 1\right) \left($
	arguments passed to other functions

## Author(s)

Vito M.R. Muggeo

### See Also

```
summary.segmented, print.summary.segmented
```

```
print.segmented.lme Print method for the segmented.lme class
```

# **Description**

Printing and extracting the most important features of a segmented mixed model.

## Usage

```
## S3 method for class 'segmented.lme'
print(x, digits = max(3, getOption("digits") - 3), ...)
## S3 method for class 'segmented.lme'
fixef(object, ...)
## S3 method for class 'segmented.lme'
logLik(object, ...)
```

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# Arguments

Χ	object of class segmented.lme
digits	number of digits to be printed
object	object of class segmented

... arguments passed to other functions

## Author(s)

Vito M.R. Muggeo

## See Also

```
segmented.lme, summary.segmented.lme
```

pscore.test

Testing for existence of one breakpoint

# Description

Given a (generalized) linear model, the (pseudo) Score statistic tests for the existence of one breakpoint.

# Usage

```
pscore.test(obj, seg.Z, k = 10, alternative = c("two.sided", "less", "greater"),
    values=NULL, dispersion=NULL, df.t=NULL, more.break=FALSE, n.break=1,
    only.term=FALSE, break.type=c("break","jump"))
```

## **Arguments**

obj	a fitted model typically returned by ${\tt glm}$ or ${\tt lm}$ . Even an object returned by segmented can be set. Offset and weights are allowed.
seg.Z	a formula with no response variable, such as seg.Z=~x1, indicating the (continuous) segmented variable being tested. Only a single variable may be tested and an error is printed when seg.Z includes two or more terms. seg.Z can be omitted if i)obj is a segmented fit with a single segmented covariate (and that variable is taken), or ii)if it is a "lm" or "glm" fit with a single covariate (and that variable is taken).
k	optional. Number of points (equi-spaced from the min to max) used to compute the pseudo Score statistic. See Details.
alternative	a character string specifying the alternative hypothesis (relevant to the slope difference parameter).
values	optional. The evaluation points where the Score test is computed. See Details for default values.

pscore.test 39

dispersion optional. the dispersion parameter for the family to be used to compute the test statistic. When NULL (the default), it is inferred from obj. Namely it is taken as 1 for the Binomial and Poisson families, and otherwise estimated by the residual Chi-squared statistic in the model obj (calculated from cases with non-zero weights divided by the residual degrees of freedom). df.t optional. The degress-of-freedom used to compute the p-value. When NULL, the df extracted from obj are used. optional, logical. If obj is a 'segmented' fit, more.break=FALSE tests for the more.break actual breakpoint for the variable 'seg,Z', while more.break=TRUE tests for an additional breakpoint(s) for the variable 'seg.Z'. Ignored when obj is not a segmented fit. n.break optional. Number of breakpoints postuled under the alternative hypothesis. only.term logical. If TRUE, only the pseudo covariate(s) relevant to the testing for the breakpoint is returned, and no test is computed. break.type The kind of breakpoint being tested. "break" is for piecewise-linear relationships, "jump" means piecewise-constant, i.e. a step-function, relationships.

#### **Details**

pscore. test tests for a non-zero difference-in-slope parameter of a segmented relationship. Namely, the null hypothesis is  $H_0: \beta=0$ , where  $\beta$  is the difference-in-slopes, i.e. the coefficient of the segmented function  $\beta(x-\psi)_+$ . The hypothesis of interest  $\beta=0$  means no breakpoint. Simulation studies have shown that such Score test is more powerful than the Davies test (see reference) when the alternative hypothesis is 'one changepoint'. If there are two or more breakpoints (for instance, a sinusoidal-like relationships), pscore.test can have lower power, and davies.test can perform better.

The dispersion value, if unspecified, is taken from obj. If obj represents the fit under the null hypothesis (no changepoint), the dispersion parameter estimate will be usually larger, leading to a (potentially severe) loss of power.

The k evaluation points are k equally spaced values in the range of the segmented covariate. k should not be small. Specific values can be set via values, although I have found no important difference due to number and location of the evaluation points, thus default is k=10 equally-spaced points. However, when the possible breakpoint is believed to lie into a specified narrower range, the user can specify k values in that range leading to higher power in detecting it, i.e. typically lower p-value.

If obj is a (segmented) lm object, the returned p-value comes from the t-distribution with appropriate degrees of freedom. Otherwise, namely if obj is a (segmented) glm object, the p-value is computed wrt the Normal distribution.

## Value

A list with class 'htest' containing the following components:

method title (character)

data.name the regression model and the segmented variable being tested

statistic the empirical value of the statistic

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parameter number of evaluation points

p.value the p-value

process the alternative hypothesis set

#### Author(s)

Vito M.R. Muggeo

#### References

Muggeo, V.M.R. (2016) Testing with a nuisance parameter present only under the alternative: a score-based approach with application to segmented modelling. *J of Statistical Computation and Simulation*, **86**, 3059–3067.

### See Also

See also davies. test.

## **Examples**

```
## Not run:
set.seed(20)
z<-runif(100)
x<-rnorm(100,2)
y<-2+10*pmax(z-.5,0)+rnorm(100,0,3)

o<-lm(y~z+x)

#testing for one changepoint
#use the simple null fit
pscore.test(o,~z) #compare with davies.test(o,~z)..

#use the segmented fit
os<-segmented(o, ~z)
pscore.test(os,~z) #smaller p-value, as it uses the dispersion under the alternative (from 'os')

#test for the 2nd breakpoint in the variable z
pscore.test(os,~z, more.break=TRUE)

## End(Not run)</pre>
```

pwr.seg

Power Analysis in segmented regression

## **Description**

Given the appropriate input values, the function computes the power (sample size) corresponding to the specifed sample size (power). If a segmented fit object is provided, the power is computed taking the parameter estimates as input values.

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## Usage

```
pwr.seg(oseg, pow, n, z = "1:n/n", psi, d, s, n.range = c(10,300), 
 X = NULL, break.type=c("break","jump"), alpha = 0.01, round.n = TRUE, alternative = c("two.sided", "greater", "less"), msg = TRUE, ci.pow=0)
```

# Arguments

oseg	The fitted segmented object. If provided, the power is computed at the model parameter estimates, and all the remaining arguments but alternative and alpha are ignored.
pow	The desired power level. If provided n has to be missing
n	The fixed sample size. If provided pow has to be missing
Z	The covariate understood to have a segmented effect. Default is "1:n/n", i.e. equispaced values in $(0,1)$ . More generally a string indicating the quantile function having p and possible other numerical values as arguments. For istance "qunif(p,0,1)", "qnorm(p,2,5)", or "qexp(p)". "qunif(p,1,n)" can be also specified, but attention should be paid to guarantee psi within the covariate range. Finally, it could be also a numerical vector meaning the actual covariate, but pow has to be missing. Namely if the covariate is supplied (and n is known), only the relevant power can be estimated.
psi	The breakpoint value within the covariate range
d	The slope difference
S	The response standard deviation
n.range	When pow is provided and the relevant sample size estimate has to be returned, the function evaluates 50 sample sizes equally spaced in n.range. However the function can also compute, via spline interpolation, sample sizes outside the specified range.
X	The design matrix including additional linear variables in the regression equation. Default to NULL which means intercept and linear term for the segmented covariate.
break.type	Type of breakpoint. break.type='break' means piecewise linear (segmented), break.type='jump' refers to piecewise constant.
alpha	The type-I error probability. Default to 0.01.
round.n	logical. If TRUE the (possible) returned sample size value is rounded.
alternative	a character string specifying the alternative hypothesis, must be one of "two.sided", "greater" or "less". Note, this refers to the sign of the slope difference.
msg	logical. If TRUE the output is returned along with a simple message, otherwise only the values are returned
ci.pow	Numerical. If oseg has been supplied, ci.pow replicates are drawn to build a 95% confidence interval for the power.

# **Details**

The function exploits the sampling distribution of the pseudo Score statistic under the alternative hypothesis of one breakpoint.

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#### Value

The computed power *or* sample size, with or without message (depending on msg)

#### Note

Currently the function assumes just 1 breakpoint in one covariate

### Author(s)

Nicoletta D'Angelo and Vito Muggeo

#### References

D'Angelo N, Muggeo V.M.R. (2021) Power analysis in segmented regression, working paper https://www.researchgate.net/publication/355885747.

Muggeo, V.M.R. (2016) Testing with a nuisance parameter present only under the alternative: a score-based approach with application to segmented modelling. *J of Statistical Computation and Simulation*, **86**, 3059–3067.

#### See Also

```
pscore.test
```

## **Examples**

```
## pwr.seg(pow=.7, psi=.5, d=1.5, s=.5) #returns the sample size
## pwr.seg(n=219, psi=.5, d=1.5, s=.5) #returns the power
## pwr.seg(n=20,z="qnorm(p, 2,5)", psi=3, d=.5, s=2) #the covariate is N(2,5)
## pwr.seg(n=20,z="qexp(p)", psi=.1, d=.5, s=.1) #the covariate is Exp(1)
```

seg

Specifying a segmented/stepmented term in the segreg/stepreg formula

## **Description**

Function used to define a segmented (stepmented) term within the segreg (stepreg) formula. The function simply passes relevant information to proper fitter functions.

## Usage

```
seg(x, npsi = 1, psi = NA, est = NA, R = NA, fixed.psi = NULL, by = NULL, f.x = I)
```

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#### **Arguments**

The segmented/stepmented (numeric) covariate Χ The number of breakpoints/jumpoints to estimate. Default to npsi=1. If by has npsi been specified, the same npsi applies to all categories of the factor by; otherwise it can be vector, wherein the entries represent the number of breakpoints of the segmented relationships within the categories of by. If npsi is specified as a vector, the corresponding est should be a list. The npsi starting values are computed according the specification of quant in seg. control. Numerical vector indicating possible starting value(s) for the breakpoint(s). When psi provided, psi overwrites npsi. If by has been specified, psi can be a list, wherein the components represent the starting values of the segmented/stepmented relationships within the categories of by. est Possible vector (of length equal to npsi+1) of 0's and 1's to indicate whether the slopes have to be estimated or fixed to zero. NA, the default, means all ones, namely every slope is estimated. Consecutive zeroes are not allowed. If by has been specified, the same est applies to all categories of the factor by; otherwise est can be a list, wherein the components represent the slope constraints of the segmented relationships within the categories of by. Note that, when a grouping variable has been specified in by, npsi can be specified as a vector, and est should be a list. R Matrix to constrain the slopes. If provided, it overwrites the matrix (which is built internally) coming from the specification of est. Possible fixed breakpoint values to be accounted for in addition to the estimated fixed.psi ones; slope and plot. segmented will account for them correctly. A possible factor meaning an interaction with the segmented term x. Hence, if by specified, a different segmented relationship is fitted within each category of by. f.x an optional function meaning a function to apply to the covariate before fitting

#### Details

The function is used within segreg and stepreg to 'build' information about the segmented relationships to fit.

## Value

The function simply returns the covariate with added attributes relevant to segmented term

## Note

If any value is provided in fix.psi, the corresponding slope difference coefficient will be labelled by \*.fixed.\*. The slope function will compute the 'right' slopes also accounting for the fixed breakpoints.

## Author(s)

Vito Muggeo

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## See Also

segreg

#### **Examples**

##see ?segreg

seg.control

Auxiliary for controlling segmented/stepmented model fitting

## Description

Auxiliary function as user interface for 'segmented' and 'stepmented' fitting. Typically only used when calling any 'segmented' or 'stepmented' method.

### Usage

```
seg.control(n.boot=10, display = FALSE, tol = 1e-05, it.max = 30, fix.npsi=TRUE,
   K = 10, quant = FALSE, maxit.glm = NULL, h = 1.25, break.boot=5, size.boot=NULL,
   jt=FALSE, nonParam=TRUE, random=TRUE, seed=NULL, fn.obj=NULL, digits=NULL,
   alpha = NULL, fc=.95, check.next=TRUE, tol.opt=NULL, fit.psi0=NULL, eta=NULL,
   min.nj=2)
```

### **Arguments**

n.boot

number of bootstrap samples used in the bootstrap restarting algorithm. If 0 the standard algorithm, i.e. without bootstrap restart, is used. Default to 10 that appears to be sufficient in most of problems. However when multiple breakpoints have to be estimated it is suggested to increase n.boot, e.g. n.boot=50, and even break.boot.

display

logical indicating if the value of the objective function should be printed along with current breakpoint estimates at each iteration or at each bootstrap resample (but no more than 5 breakpoints are printed). If bootstrap restarting is employed, the values of objective and breakpoint estimates should not change at the last runs

tol

positive convergence tolerance.

it.max

integer giving the maximal number of iterations.

fix.npsi

logical (it replaces previous argument stop.if.error) If TRUE (default) the *number* (and not location) of breakpoints is held fixed throughout iterations. Otherwise a sort of 'automatic' breakpoint selection is carried out, provided that several starting values are supplied for the breakpoints, see argument psi in segmented.lm or segmented.glm. The idea, relying on removing the 'non-admissible' breakpoint estimates at each iteration, is discussed in Muggeo and Adelfio (2011) and it is not compatible with the bootstrap restart algorithm. fix.npsi=FALSE, indeed, should be considered as a preliminary and tentative approach to deal with an unknown number of breakpoints. Alternatively, see selgmented.

seg.control 45

K the number of quantiles (or equally-spaced values) to supply as starting values for the breakpoints when the psi argument of segmented is set to NA. K is ignored when psi is different from NA.

logical, indicating how the starting values should be selected. If FALSE equally-spaced values are used, otherwise the quantiles. Ignored when psi is different from NA.

integer giving the maximum number of inner IWLS iterations (see details). If NULL, the number is low in the first iterations and then increases as the process goes on. Ignored for segmented lm fits

positive factor modifying the increments in breakpoint updates during the estimation process (see details).

Integer, less than n.boot. If break.boot consecutive bootstrap samples lead to the same objective function during the estimation process, the algorithm stops without performing all n.boot 'trials'. This can save computational time considerably. Default is 5 for the segmented and 5+2 for the stepmented functions. However if the number of changepoints is large, break.boot should be increased, even 10 or 15.

the size of the bootstrap samples. If NULL, it is taken equal to the actual sample size. If the sample is very large, the idea is to run bootstrap restarting using smaller bootstrap samples.

logical. If TRUE the values of the segmented variable(s) are jittered before fitting the model to the bootstrap resamples.

if TRUE nonparametric bootstrap (i.e. case-resampling) is used, otherwise residual-based. Currently working only for LM fits. It is not clear what residuals should be used for GLMs.

if TRUE, when the algorithm fails to obtain a solution, random values are employed to obtain candidate values.

The seed to be passed on to set.seed() when n.boot>0. If NULL, a seed depending on the response values is generated and used. Otherwise it can be a numerical value or, if NA, a random value is generated. Fixing the seed can be useful to replicate *exactly* the results when the bootstrap restart algorithm is employed. Whichever choice, the segmented fit includes the component seed representing the value saved just before the bootstrap resampling. Re-use it if you want to replicate the bootstrap restarting algorithm with the *same* re-samples.

A character string to be used (optionally) only when segmented.default is used. It represents the function (with argument 'x') to be applied to the fit object to extract the objective function to be minimized. Thus for "lm" fits (although unnecessary) it should be fn.obj="sum(x\$residuals^2)", for "coxph" fits it should be fn.obj="-x\$loglik[2]". If NULL the 'minus log likelihood' extracted from the object, namely "-logLik(x)", is used. See segmented.default.

optional. If specified it means the desidered number of decimal points of the breakpoint to be used during the iterative algorithm.

optional numerical values. The breakpoints are estimated within the quantiles alpha[1] and alpha[2] of the relevant covariate. If a single value is provided, it is assumed alpha and 1-alpha. Defaults to NULL which means alpha=max(.05,

quant

h

break.boot

maxit.glm

size.boot

jt

nonParam

random

seed

fn.obj

digits

alpha

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	1/n). Note: Providing alpha=c(mean(x<=a),mean(x<=b)) means to constrain the breakpoint estimates within $[a,b]$ .
fc	A proportionality factor $(\leq 1)$ to adjust the breakpoint estimates $if$ these come close to the boundary or too close each other. For instance, if psi turns up close to the maximum, it will be changed to psi*fc or to psi/fc if close to the minimum. This is useful to get finite point estimate and standard errors for each slope paramete.
check.next	logical, effective only for stepmented fit. If TRUE the solutions next to the current one are also investigated.
tol.opt	Numerical value to be passed to tol in optimize.
fit.psi0	Possible list including preliminary values.
eta	Only for segmented/stepmented fits: starting values to be passed to etastart in ${\tt glm.fit.}$
min.nj	How many observations (at least) should be in the covariate intervals induced by the breakpoints?

#### **Details**

Fitting a 'segmented' GLM model is attained via fitting iteratively standard GLMs. The number of (outer) iterations is governed by it.max, while the (maximum) number of (inner) iterations to fit the GLM at each fixed value of psi is fixed via maxit.glm. Usually three-four inner iterations may be sufficient.

When the starting value for the breakpoints is set to NA for any segmented variable specified in seg. Z, K values (quantiles or equally-spaced) are selected as starting values for the breakpoints.

Since version 0.2-9.0 segmented implements the bootstrap restarting algorithm described in Wood (2001). The bootstrap restarting is expected to escape the local optima of the objective function when the segmented relationship is noisy and the loglikelihood can be flat. Notice bootstrap restart runs n.boot iterations regardless of tol that only affects convergence within the inner loop.

## Value

A list with the arguments as components.

## Author(s)

Vito Muggeo

## References

Muggeo, V.M.R., Adelfio, G. (2011) Efficient change point detection in genomic sequences of continuous measurements. *Bioinformatics* **27**, 161–166.

Wood, S. N. (2001) Minimizing model fitting objectives that contain spurious local minima by bootstrap restarting. *Biometrics* **57**, 240–244.

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## **Examples**

#decrease the maximum number inner iterations and display the
#evolution of the (outer) iterations
#seg.control(display = TRUE, maxit.glm=4)

seg.lm.fit

Fitter Functions for Segmented Linear Models

### **Description**

seg.lm.fit is called by segmented.lm to fit segmented linear (gaussian) models. Likewise, seg.glm.fit is called by segmented.glm to fit generalized segmented linear models, and seg.def.fit is called by segmented.default to fit segmented relationships in general regression models (e.g., quantile regression and Cox regression). seg.lm.fit.boot, seg.glm.fit.boot, and seg.def.fit.boot are employed to perform bootstrap restart. The functions segConstr.\* are called by segreg() when some contraints are set on the slopes of the segmented relationships.

These functions should usually not be used directly by the user.

## Usage

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```
segConstr.lm.fit(y, XREG, Z, PSI, w, offs, opz, return.all.sol = FALSE)
segConstr.lm.fit.boot(y, XREG, Z, PSI, w, offs, opz, n.boot=10, size.boot=NULL,
    jt=FALSE, nonParam=TRUE, random=FALSE, break.boot=n.boot)
segConstr.glm.fit(y, XREG, Z, PSI, w, offs, opz, return.all.sol = FALSE)
segConstr.glm.fit.boot(y, XREG, Z, PSI, w, offs, opz, n.boot=10, size.boot=NULL,
    jt=FALSE, nonParam=TRUE, random=FALSE, break.boot=n.boot)
```

## **Arguments**

У	vector of observations of length n.
XREG	design matrix for standard linear terms.
Z	appropriate matrix including the segmented variables whose breakpoints have to be estimated.
PSI	appropriate matrix including the starting values of the breakpoints to be estimated.
W	possibe weights vector.
offs	possibe offset vector.
opz	a list including information useful for model fitting.
n.boot	the number of bootstrap samples employed in the bootstrap restart algorithm.
break.boot	Integer, less than n.boot. If break.boot consecutive bootstrap samples lead to the same objective function, the algorithm stops without performing all n.boot 'trials'. This can save computational time considerably.
size.boot	the size of the bootstrap resamples. If NULL (default), it is taken equal to the sample size. values smaller than the sample size are expected to increase perturbation in the bootstrap resamples.
jt	logical. If TRUE the values of the segmented variable(s) are jittered before fitting the model to the bootstrap resamples.
nonParam	if TRUE nonparametric bootstrap (i.e. case-resampling) is used, otherwise residual-based.
random	if TRUE, when the algorithm fails to obtain a solution, random values are used as candidate values.
return.all.sol	if TRUE, when the algorithm fails to obtain a solution, the values visited by the algorithm with corresponding deviances are returned.
obj	the starting regression model where the segmented relationships have to be added.
mfExt	the model frame.

## **Details**

The functions call iteratively lm.wfit (or glm.fit) with proper design matrix depending on XREG, Z and PSI. seg.lm.fit.boot (and seg.glm.fit.boot) implements the bootstrap restarting idea discussed in Wood (2001).

### Value

A list of fit information.

#### Note

These functions should usually not be used directly by the user.

### Author(s)

Vito Muggeo

### References

Wood, S. N. (2001) Minimizing model fitting objectives that contain spurious local minima by bootstrap restarting. *Biometrics* **57**, 240–244.

### See Also

```
segmented.lm, segmented.glm
```

### **Examples**

##See ?segmented

segmented

Segmented relationships in regression models

## **Description**

Fits regression models with segmented relationships between the response and one or more explanatory variables. Break-point estimates are provided.

## Usage

```
segmented(obj, seg.Z, psi, npsi, fixed.psi=NULL, control = seg.control(),
    model = TRUE, ...)

## Default S3 method:
segmented(obj, seg.Z, psi, npsi, fixed.psi=NULL, control = seg.control(),
    model = TRUE, keep.class=FALSE, ...)

## S3 method for class 'lm'
segmented(obj, seg.Z, psi, npsi, fixed.psi=NULL, control = seg.control(),
    model = TRUE, keep.class=FALSE, ...)

## S3 method for class 'glm'
segmented(obj, seg.Z, psi, npsi, fixed.psi=NULL, control = seg.control(),
    model = TRUE, keep.class=FALSE, ...)
```

```
## S3 method for class 'Arima'
segmented(obj, seg.Z, psi, npsi, fixed.psi=NULL, control = seg.control(),
    model = TRUE, keep.class=FALSE, ...)

## S3 method for class 'numeric'
segmented(obj, seg.Z, psi, npsi, fixed.psi=NULL, control = seg.control(),
    model = TRUE, keep.class=FALSE, adjX=FALSE, weights=NULL, ...)
```

### **Arguments**

obj

standard 'linear' model of class "lm", "glm" or "Arima", or potentially any regression fit may be supplied since version 0.5-0 (see 'Details'). obj can include any covariate understood to have a linear (i.e. no break-points) effect on the response. If obj also includes the segmented covariate specified in seg. Z, then all the slopes of the fitted segmented relationship will be estimated. On the other hand, if obj misses the segmented variable, then the 1st (the leftmost) slope is assumed to be zero. Since version 1.5.0, obj can be a simple numeric or ts object but with only a single segmented variable (segmented.numeric) see examples below.

seg.Z

the segmented variable(s), i.e. the continuous covariate(s) understood to have a piecewise-linear relationship with response. It is a formula with no response variable, such as seg.Z=~x or seg.Z=~x1+x2. It can be missing when obj includes only one covariate which is taken as segmented variable. Currently, formulas involving functions, such as seg.Z=~log(x1), or selection operators, such as seg.Z=~d[,"x1"] or seg.Z=~d\$x1, are *not* allowed. Also, variable names formed by U or V only (with or without numbers) are not permitted.

psi

starting values for the breakpoints to be estimated. If there is a single segmented variable specified in seg.Z, psi is a numeric vector, and it can be missing when 1 breakpoint has to be estimated (and the median of the segmented variable is used as a starting value). If seg.Z includes several covariates, psi has be specified as a *named* list of vectors whose names have to match the variables in the seg.Z argument. Each vector of such list includes starting values for the break-point(s) for the corresponding variable in seg.Z. A NA value means that 'K' quantiles (or equally spaced values) are used as starting values; K is fixed via the seg.control auxiliary function. See npsi as an alternative to specify just the number of breakpoints.

npsi

A named vector or list meaning the *number* (and not locations) of breakpoints to be estimated. The starting values will be internally computed via the quantiles or equally spaced values, as specified in argument quant in seg.control. npsi can be missing and npsi=1 is assumed for all variables specified in seg.Z. If psi is provided, npsi is ignored.

fixed.psi

An optional named list meaning the breakpoints to be kept fixed during the estimation procedure. The names should be a subset of (or even the same) variables specified in seg.Z. If there is a single variable in seg.Z, a simple numeric vector can be specified. Note that, in addition to the values specified here, segmented will estimate additional breakpoints. To keep fixed all breakpoints (to be specified in psi) use it.max=0 in seg.control

control a list of parameters for controlling the fitting process. See the documentation for seg.control for details. logical value indicating if the model.frame should be returned. model keep.class logical value indicating if the final fit returned by segmented.default should keep the class 'segmented' (along with the class of the original fit obj). Ignored by the segmented methods. optional arguments (to be ignored safely). Notice specific arguments relevant to the original call (via 1m or glm for instance), such as weights or offet, have to be included in the starting model obj adiX if obj is a ts, the segmented variable (if not specified in seg. Z) is computed by taking information from the time series (e.g., years starting from 2000, say). If adjX=TRUE, the segmented variable is shifted such that its min equals zero. Default is using the unshifted values, but if there are several breakpoints to be estimated, it is strongly suggested to set adjX=TRUE. the weights if obj is a vector or a ts object, otherwise the weights should be weights

specified in the starting fit obj.

#### **Details**

Given a linear regression model usually of class "lm" or "glm" (or even a simple numeric/ts vector), segmented tries to estimate a new regression model having broken-line relationships with the variables specified in seg. Z. A segmented (or broken-line) relationship is defined by the slope parameters and the break-points where the linear relation changes. The number of breakpoints of each segmented relationship is fixed via the psi (or npsi) argument, where initial values for the breakpoints (or simply their number via npsi) must be specified. The model is estimated simultaneously yielding point estimates and relevant approximate standard errors of all the model parameters, including the break-points.

Since version 0.2-9.0 segmented implements the bootstrap restarting algorithm described in Wood (2001). The bootstrap restarting is expected to escape the local optima of the objective function when the segmented relationship is flat and the log likelihood can have multiple local optima.

Since version 0.5-0.0 the default method segmented.default has been added to estimate segmented relationships in general (besides "Im" and "glm" fits) regression models, such as Cox regression or quantile regression (for a single percentile). The objective function to be minimized is the (minus) value extracted by the logLik function or it may be passed on via the fn.obj argument in seg.control. See example below. While the default method is expected to work with any regression fit (where the usual coef(), update(), and logLik() returns appropriate results), it is not recommended for "Im" or "glm" fits (as segmented.default is slower than the specific methods segmented.lm and segmented.glm), although final results are the same. However the object returned by segmented default is *not* of class "segmented", as currently the segmented methods are not guaranteed to work for 'generic' (i.e., besides "Im" and "glm") regression fits. The user could try each "segmented" method on the returned object by calling it explicitly (e.g. via plot. segmented() or confint.segmented() wherein the regression coefficients and relevant covariance matrix have to be specified, see .coef and .vcov in plot.segmented(), confint.segmented(), slope()).

#### Value

segmented returns an object of class "segmented" which inherits from the class of obj, for instance "lm" or "glm".

An object of class "segmented" is a list containing the components of the original object obj with additionally the followings:

psi estimated break-points (sorted) and relevant (approximate) standard errors

it number of iterations employed

epsilon difference in the objective function when the algorithm stops

model the model frame

psi.history a list or a vector including the breakpoint estimates at each step

seed the integer vector containing the seed just before the bootstrap resampling. Re-

turned only if bootstrap restart is employed

.. Other components are not of direct interest of the user

### Warning

At convergence, if the estimated breakpoints are too close each other or at the boundaries, the parameter point estimate could be returned, but without finite standard errors. To avoid that, segmented revises the final breakpoint estimates to allow that at least min.nj are within each interval of the segmented covariate. A warning message is printed if such adjustment is made. See min.nj in seg.control.

#### Note

1. The algorithm will start if the it.max argument returned by seg.control is greater than zero. If it.max=0 segmented will estimate a new linear model with break-point(s) fixed at the values reported in psi.Alternatively, it is also possible to set h=0 in seg.control(). In this case, bootstrap restarting is unnessary, then to have breakpoints at mypsi type

```
segmented(.., psi=mypsi, control=seg.control(h=0, n.boot=0, it.max=1))
```

- 2. In the returned fit object, 'U.' is put before the name of the segmented variable to mean the difference-in-slopes coefficient.
- 3. Methods specific to the class "segmented" are
  - print.segmented
  - summary.segmented
  - print.summary.segmented
  - plot.segmented
  - lines.segmented
  - confint.segmented
  - vcov.segmented
  - predict.segmented
  - points.segmented
  - coef.segmented

Others are inherited from the class "lm" or "glm" depending on the class of obj.

### Author(s)

```
Vito M. R. Muggeo, <vito.muggeo@unipa.it>
```

#### References

Muggeo, V.M.R. (2003) Estimating regression models with unknown break-points. *Statistics in Medicine* **22**, 3055–3071.

Muggeo, V.M.R. (2008) Segmented: an R package to fit regression models with broken-line relationships. *R News* **8/1**, 20–25.

#### See Also

segmented.glm for segmented GLM and segreg to fit the models via a formula interface. segmented.lme fits random changepoints (segmented mixed) models.

## **Examples**

```
set.seed(12)
xx<-1:100
zz < -runif(100)
yy<-2+1.5*pmax(xx-35,0)-1.5*pmax(xx-70,0)+15*pmax(zz-.5,0)+rnorm(100,0,2)
dati<-data.frame(x=xx,y=yy,z=zz)</pre>
out.lm < -lm(y \sim x, data = dati)
#the simplest example: the starting model includes just 1 covariate
#.. and 1 breakpoint has to be estimated for that
o<-segmented(out.lm) #1 breakpoint for x
#the single segmented variable is not in the starting model, and thus..
#... you need to specify it via seg.Z, but no starting value for psi
o<-segmented(out.lm, seg.Z=~z)
#note the leftmost slope is constrained to be zero (since out.lm does not include z)
#2 segmented variables, 1 breakpoint each (again no need to specify npsi or psi)
o < -segmented(out.lm, seg.Z = ~z + x)
#1 segmented variable, but 2 breakpoints: you have to specify starting values (vector) for psi:
o<-segmented(out.lm,seg.Z=~x,psi=c(30,60), control=seg.control(display=FALSE))
#.. or you can specify just the *number* of breakpoints
#o<-segmented(out.lm,seg.Z=~x, npsi=2, control=seg.control(display=FALSE))</pre>
slope(o) #the slopes of the segmented relationship
#2 segmented variables: starting values requested via a named list
out.lm < -lm(y \sim z, data = dati)
o1<-update(o,seg.Z=~x+z,psi=list(x=c(30,60),z=.3))
#..or by specifying just the *number* of breakpoints
\#o1 < -update(o, seg.Z=^x+z, npsi=c(x=2, z=1))
```

```
#the default method leads to the same results (but it is slower)
\#o1 < -\text{segmented.default}(\text{out.lm}, \text{seg.Z} = -\text{x+z}, \text{psi=list}(\text{x=c}(30,60), \text{z=.3}))
#o1<-segmented.default(out.lm,seg.Z=~x+z,psi=list(x=c(30,60),z=.3),</pre>
     control=seg.control(fn.obj="sum(x$residuals^2)"))
#automatic procedure to estimate breakpoints in the covariate x (starting from K quantiles)
# Hint: increases number of iterations. Notice: bootstrap restart is not allowed!
# However see ?selgmented for a better approach
#o<-segmented.lm(out.lm, seg.Z=~x+z, psi=list(x=NA, z=.3),</pre>
# control=seg.control(fix.npsi=FALSE, n.boot=0, tol=1e-7, it.max = 50, K=5, display=TRUE))
#assess the progress of the breakpoint estimates throughout the iterations
## Not run:
par(mfrow=c(1,2))
draw.history(o, "x")
draw.history(o, "z")
## End(Not run)
#try to increase the number of iterations and re-assess the
#convergence diagnostics
# A simple segmented model with continuous responses and no linear covariates
# No need to fit the starting lm model:
segmented(yy, npsi=2) #NOTE: subsetting the vector works ( segmented(yy[-1],..) )
#only a single segmented covariate is allowed in seg.Z, and if seg.Z is unspecified,
# the segmented variable is taken as 1:n/n
# An example using the Arima method:
## Not run:
n<-50
idt <-1:n #the time index
mu < -50 - idt + 1.5 * pmax(idt - 30,0)
set.seed(6969)
y<-mu+arima.sim(list(ar=.5),n)*3.5
o < -arima(y, c(1,0,0), xreg=idt)
os1<-segmented(o, ~idt, control=seg.control(display=TRUE))
#note using the .coef argument is mandatory!
slope(os1, .coef=os1$coef)
plot(y)
plot(os1, add=TRUE, .coef=os1$coef, col=2)
## End(Not run)
```

```
#####Four examples using the default method:
#==> 1. Cox regression with a segmented relationship
## Not run:
library(survival)
data(stanford2)
o<-coxph(Surv(time, status)~age, data=stanford2)
os<-segmented(o, ~age, psi=40) #estimate the breakpoint in the age effect
summary(os) #actually it means summary.coxph(os)
plot(os) #it does not work
plot.segmented(os) #call explicitly plot.segmented() to plot the fitted piecewise lines
# ==> 2. Linear mixed model via the nlme package
dati$g<-gl(10,10) #the cluster 'id' variable
library(nlme)
o<-lme(y~x+z, random=~1|g, data=dati)
os<-segmented.default(o, \sim x+z, npsi=list(x=2, z=1))
#summarizing results (note the '.coef' argument)
slope(os, .coef=fixef(os))
plot.segmented(os, "x", .coef=fixef(os), conf.level=.95)
confint.segmented(os, "x", .coef=fixef(os))
dd < -data.frame(x=c(20,50),z=c(.2,.6), g=1:2)
predict.segmented(os, newdata=dd, .coef=fixef(os))
# ==> 3. segmented quantile regression via the quantreg package
library(quantreg)
data(Mammals)
y<-with(Mammals, log(speed))</pre>
x<-with(Mammals, log(weight))</pre>
o < -rq(y \sim x, tau = .9)
os<-segmented.default(o, ~x) #it does NOT work. It cannot compute the vcov matrix..
#Let's define the vcov.rq function.. (I don't know if it is the best option..)
vcov.rq<-function(x,...) {</pre>
 V<-summary(x,cov=TRUE,se="nid",...)$cov</pre>
 rownames(V)<-colnames(V)<-names(x$coef)</pre>
```

```
٧}
os<-segmented.default(o, ~x) #now it does work
plot.segmented(os, res=TRUE, col=2, conf.level=.95)
# ==> 4. segmented regression with the svyglm() (survey package)
library(survey)
data(api)
dstrat<-svydesign(id=~1,strata=~stype, weights=~pw, data=apistrat, fpc=~fpc)
o<-svyglm(api00~ell, design=dstrat)
#specify as a string the objective function to be minimized. It can be obtained via svyvar()
fn.x<- 'as.numeric(svyvar(resid(x, "pearson"), x$survey.design, na.rm = TRUE))'</pre>
os<-segmented.default(o, ~ell, control=seg.control(fn.obj=fn.x, display=TRUE))
slope(os)
plot.segmented(os, res=TRUE, conf.level=.9, shade=TRUE)
## End(Not run)
```

segmented.lme

Segmented relationships in linear mixed models

### Description

Fits linear mixed models with a segmented relationship between the response and a numeric covariate. Random effects are allowed in each model parameter, including the breakpoint.

### Usage

```
## S3 method for class 'lme'
segmented(obj, seg.Z, psi, npsi = 1, fixed.psi = NULL,
    control = seg.control(), model = TRUE,
    z.psi = ~1, x.diff = ~1, random = NULL,
    random.noG = NULL, start.pd = NULL, psi.link = c("identity", "logit"),
    start = NULL, data, fixed.parms = NULL,...)
```

## **Arguments**

obj A 'lme' fit returned by 1me or simply its call. See example below. This represents the linear mixed model where the segmented relationship is added.

seg. Z A one-sided formula indicating the segmented variable, i.e. the quantitative variable having a segmented relationship with the response. In longitudinal studies typically it is the time.

psi	An optional starting value for the breakpoint. If missing a starting value is obtained via the nadir estimate of a quadratic fit. When provided it may be a single numeric value or a vector of length equal to the number of clusters (i.e. subjects).
z.psi	Optional. A one-sided formula meaning the covariates in the sub-regression model for the changepoint parameter. Default to ~1.
x.diff	Optional. A one-sided formula meaning the covariates in the sub-regression model for the difference-in-slopes parameter. Default to ~1 for no covariate for the difference-in-slope parameter.
npsi	Ignored. Currently only npsi=1 is allowed.
fixed.psi	Ignored.
control	A list returned by seg.control, in particular display, n. boot for the bootstrap restarting.
model	Ignored.
random	A list, as the one supplied in random of lme() including the random effects. Default to NULL, meaning that the same random effect structure of the initial lme fit supplied in obj should be used. When specified, this list could include the variables 'G0' and 'U'. G0 means random effects in the breakpoints and U means random effects in the slope-difference parameter. Assuming id is the the cluster variable and x the segmented variable, some examples are
	random = list(id = pdDiag(~1 + x + U)) #ind. random eff. (changepoint fixed)
	random = list(id = pdDiag( $^{-1}$ + x + U + G0)) #ind. random eff. (in the change-point too)
	<pre>random = list(id=pdBlocked(list(pdSymm(~1+x), pdSymm(~U+G0-1)))) #block diagonal</pre>
random.noG	Ignored.
start.pd	An optional starting value for the variances of the random effects. It should be coherent with the specification in random.
psi.link	The link function used to specify the sub-regression model for the breakpoint <i>psi</i> . The identity (default) assumes

psi. The identity (default) assumes

$$\psi_i = \eta_i$$

while the logit link is

$$\psi_i = (m + M * exp(\eta_i))/(1 + exp(\eta_i))$$

where m and M are the observed minimum and maximum of the segmented variable in seg. Z. In each case the 'linear predictor' is  $\eta_i = \kappa_0 + z_i^T \kappa_1 + k_i$ , where  $z^T$  includes the covariates specified in z.psi and the  $k_i$ s are the changepoint random effects included by means of G0 in the random argument.

start

An optional list including the starting values for the difference-in-slopes parameter, delta0 and delta, and the changepoint parameter, kappa and kappa0. When provided, 'kappa0' overwrites 'psi'.

If provided, the components 'delta' and 'kappa' should be *named* vectors with length and names matching length and names in x.diff and z.psi respectively. The component delta0 can be a scalar or a vector with length equal to the number of clusters (subjects).

the dataframe where the variables are stored. If missing, the dataframe of the

"lme" fit obj is assumed.

fixed.parms An optional named vector representing the coefficients of the changepoint to

be maintained *fixed* during the estimation process. Allowed names are "G0" or any variable (in the dataframe) supposed to affect the location of breakpoints. For instance fixed.parms=c(G0=.3) implies a fixed value for the changepoint. Notice if you use the same variable in fixed.parms and in z.psi, for instance fixed.parms=c(x2=.3) and z.psi=x2, a warning is printed and the coefficient "G.x2" is estimated to maximize the log likelihood *given* that fixed value. As an example, suppose the unconstrained estimated coefficient for x2, say, in z.psi is 0.5; if in a new call both fixed.parms=c(x2=.4) and z.psi=x2 are included, the estimate of "G.x2" will be (approximately) 0.1. Essentially, if you really want to fix the parameters in fixed.parms, then do not include the same covariates in z.psi.

covariates in 2.ps

... Ignored

#### **Details**

The function fits segmented mixed regression models, i.e. segmented models with random effects also in the slope-difference and change-point parameters.

#### Value

A list of class segmented. Ime with several components. The most relevant are

lme.fit The fitted lme object at convergence

lme.fit.noG The fitted lme object at convergence assuming known the breakpoints

psi.i The subject/cluster-specific change points (fixed + random). It includes 2 at-

tributes: attr(,"ni") for the number of measurements in each 'cluster', and attr(,"is.break") a vector of logicals indicating if the breakpoint for each subject i can be reliable (TRUE) or not (FALSE). Here 'reliable' simply means

within the covariate range (for subject i). See also argument nq.

fixed.eta.psi The fixed-effect linear predictor for the change points regression equation. These

values will different among 'clusters' only if at least one covariate has been spec-

ified in z.psi.

fixed.eta.delta

The fixed-effect linear predictor of the slope difference regression equation. These values will different among 'clusters' only if at least one covariate has

been specified in x.diff.

#### Warning

The function deals with estimation with a *single* breakpoint only.

## Note

Currently only one breakpoint (with or without random effects) can be estimated. If fit is the segmented.lme fit, use VarCorr(fit\$lme.fit) to extract the random effect covariance matrix.

### Author(s)

Vito M.R. Muggeo <vito.muggeo@unipa.it>

#### References

Muggeo V., Atkins D.C., Gallop R.J., Dimidjian S. (2014) Segmented mixed models with random changepoints: a maximum likelihood approach with application to treatment for depression study. Statistical Modelling, 14, 293-313.

Muggeo V. (2016) Segmented mixed models with random changepoints in R. Working paper available on RG. doi: 10.13140/RG.2.1.4180.8402

## See Also

plot.segmented.lme for the plotting method and segmented.default (example 2) for segmented models with no random effects in breakpoints or slope difference.

### **Examples**

```
## Not run:
library(nlme)
data(Cefamandole)
Cefamandole$1Time <-log(Cefamandole$Time)</pre>
Cefamandole$lconc <-log(Cefamandole$conc)</pre>
o<-lme(lconc ~ lTime, random=~1|Subject, data=Cefamandole)
os<-segmented.lme(o, ~lTime, random=list(Subject=pdDiag(~1+lTime+U+G0)),
 control=seg.control(n.boot=0, display=TRUE))
slope(os)
# covariate effect on the changepoint and slope diff
#let's assume a new subject-specific covariates..
set.seed(69)
Cefamandole$z <- rep(runif(6), rep(14,6))</pre>
Cefamandole$group <- gl(2,42,labels=c('a','b'))</pre>
#Here 'group' affects the slopes and 'z' affects the changepoint
o1 <-lme(lconc ~ lTime*group, random=~1|Subject, data=Cefamandole)
os1 <- segmented(o1, ~lTime, x.diff=~group, z.psi=~z,
 random=list(Subject=pdDiag(~1+lTime+U+G0)))
slope(os1, by=list(group="a")) #the slope estimates in group="a" (baseline level)
slope(os1, by=list(group="b")) #the slope estimates in group="b"
```

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```
# A somewhat "complicated" example:
      i) strong heterogeneity in the changepoints
      ii) No changepoint for the Subject #7 (added)
d<-Cefamandole
d$x<- d$lTime
d$x[d$Subject==1]<- d$lTime[d$Subject==1]+3
d$x[d$Subject==5]<- d$lTime[d$Subject==5]+5
d$x[d$Subject==3]<- d$lTime[d$Subject==3]-5
d<-rbind(d, d[71:76,])</pre>
d$Subject <- factor(d$Subject, levels=c(levels(d$Subject),"7"))</pre>
d$Subject[85:90] <- rep("7",6)
o<-lme(lconc ~ x, random=~1|Subject, data=d)
os2<-segmented.lme(o, ~x, random=list(Subject=pdDiag(~1+x+U+G0)),
 control=seg.control(n.boot=5, display=TRUE))
#plots with common x- and y- scales (to note heterogeneity in the changepoints)
plot(os2, n.plot = c(3,3))
attr(os2$psi.i, "is.break") #it is FALSE for Subject #7
#plots with subject-specific scales
plot(os2, n.plot = c(3,3), xscale=-1, yscale = -1)
## End(Not run)
```

segreg

Fitting segmented/stepmented regression

## **Description**

segreg (stepreg) fits (generalized) linear segmented (stepmented) regression via a symbolic description of the linear predictor. This is an alternative but equivalent function, introduced since version 2.0-0 (segreg) and 2.1-0 (stepreg), to segmented. (g)lm or stepmented. (g)lm.

# Usage

```
segreg(formula, data, subset, weights, na.action, family = lm, control = seg.control(),
    transf = NULL, contrasts = NULL, model = TRUE, x = FALSE, var.psi = TRUE, ...)

stepreg(formula, data, subset, weights, na.action, family = lm, control = seg.control(),
    transf = NULL, contrasts = NULL, model = TRUE, x = FALSE, var.psi = FALSE, ...)
```

#### **Arguments**

formula

A standard model formula also including one or more 'segmented'/'stepmented' terms via the function seg

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data The possible dataframe where the variables are stored

subset Possible subset, as in 1m or glm

weights Possible weight vector, see weights in lm or glm

na.action a function which indicates what happen when the data contain NA values. See

na.action in 1m or glm.

family The family specification, similar to family in glm. Default to 'lm' for seg-

mented/stepmented linear models.

control See seg.control

transf an optional character string (with "y" as argument) meaning a function to apply

to the response variable before fitting

contrasts see contrasts in glm

model If TRUE, the model frame is returned.

x If TRUE, the model matrix is returned.

var.psi logical, meaning if the standard errors for the breakpoint estimates should be

returned in the object fit. If FALSE, the standard errors will be computed by vcov.segmented or summary.segmented. Setting var.psi=FALSE could speed up model estimation for very large datasets. Default to TRUE for segreg and

FALSE for stepreg.

... Ignored

#### **Details**

The function allows to fit segmented/stepmented (G)LM regression models using a formula interface. Results will be the same of those coming from the traditional segmented.lm and segmented.glm (or stepmented.lm or stepmented.glm), but there are some additional facilities: i) it is possible to estimate strightforwardly the segmented/stepmented relationships in each level of a categorical variable, see argument by in seg; ii) it is possible to constrain some slopes of the segmented relationship, see argument est or R in seg.

#### Value

An object of class "segmented" (or "stepmented") which inherits from the class "lm" or "glm" depending on family specification. See segmented.lm.

#### Warning

Currently for fits returned by segreg, confint.segmented only works if method="delta". Constraints on the mean levels (possibly via argument 'est' of seg) are not yet allowed when calling stepreg.

#### Note

When the formula includes even a single segmented term with constraints (specified via the argument est in seg()), the relevant coefficients returned do not represent the slope differences as in segmented.lm or segmented.glm. The values depend on the constraints and are not usually interpretable. Use slope the recover the actual slopes of the segmented relationships.

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### Author(s)

Vito Muggeo

#### References

Muggeo, V.M.R. (2003) Estimating regression models with unknown break-points. Statistics in Medicine 22, 3055-3071.

### See Also

```
seg, segmented, stepmented
```

## **Examples**

```
#####################################
#An example using segreg()
set.seed(10)
x<-1:100
z < -runif(100)
w<-runif(100,-10,-5)
y < -2+1.5*pmax(x-35,0)-1.5*pmax(x-70,0)+10*pmax(z-.5,0)+rnorm(100,0,2)
##the traditional approach
out.lm<-lm(y^x+z+w)
o<-segmented(out.lm, seg.Z=^xx+z, psi=list(x=c(30,60),z=.4))
o1<-segreg(y ~ w+seg(x,npsi=2)+seg(z))
all.equal(fitted(o), fitted(o1))
#put some constraints on the slopes
o2<-segreg(y \sim w+seg(x,npsi=2, est=c(0,1,0))+seg(z))
o3<-segreg(y ~ w+seg(x,npsi=2, est=c(0,1,0))+seg(z, est=c(0,1)))
slope(o2)
slope(o3)
##see ?plant for an additional example
#####################################
#An example using stepreg()
### Two stepmented covariates (with 1 and 2 breakpoints)
n=100
x<-1:n/n
z < -runif(n, 2, 5)
w<-rnorm(n)</pre>
mu < -2 + 1*(x > .6) - 2*(z > 3) + 3*(z > 4)
y < - mu + rnorm(n) * .8
```

```
os <-stepreg(y~seg(x)+seg(z,2)+w) #also includes 'w' as a possible linear term os summary(os) plot(os, "z", col=2:4) #plot the effect of z
```

selgmented

Selecting the number of breakpoints in segmented regression

## **Description**

This function selects (and estimates) the number of breakpoints of the segmented relationship according to the BIC/AIC criterion or sequential hypothesis testing.

## Usage

```
selgmented(olm, seg.Z, Kmax=2, type = c("score", "bic", "davies", "aic"),
   alpha = 0.05, control = seg.control(), refit = FALSE, stop.if = 5,
   return.fit = TRUE, bonferroni = FALSE, msg = TRUE, plot.ic = FALSE, th = NULL,
   G = 1, check.dslope = TRUE)
```

## **Arguments**

olm	A starting lm or glm object, or a simple numerical vector meaning the response variable.
seg.Z	A one-side formula for the segmented variable. Only one term can be included, and it can be omitted if olm is a (g)lm fit including just one numeric covariate. Also it might be omitted, and will be taken as 1,2, if olm includes a single numeric variable.
Kmax	The maximum number of breakpoints being tested. If type='bic' or type='aic' any integer value can be specified; otherwise at most Kmax=2 breakpoints can be tested via the Score or Davies statistics.
type	Which criterion should be used? Options "score" and "davies" allow to carry out sequential hypothesis testing with no more than 2 breakpoints (Kmax=2). Alternatively, the number of breakpoints can be selected via the BIC (or AIC) with virtually no upper bound for Kmax.
alpha	The fixed type I error probability when sequential hypothesis testing is carried out (i.e. type='score' or 'davies'). It is also used when type='bic' (or type='aic') and check.dslope=TRUE to remove the breakpoints based on the slope diffence t-value.
control	See seg.control.
refit	If TRUE, the final selected model is re-fitted using arguments in control, typically with bootstrap restarting. Set refit=FALSE to speed up computation (and possibly accepting near-optimal estimates). It is always TRUE if type='score' or type='davies'.

An integer. If, when trying models with an increasing (when G=1) or decreasing (when G>1) number of breakpoints, stop.if consecutive fits exhibit higher AIC/BIC values, the search is interrupted. Set a large number, larger then Kmax say, if you want to assess the fits for all breakpoints 0, 1, 2, ..., Kmax. Ignored if type='score' or type='davies'.

return.fit If TRUE, the fitted model (with the number of breakpoints selected according to type) is returned.

If TRUE, the Bonferroni correction is employed, i.e. alpha/Kmax (rather than alpha) is always taken as threshold value to reject or not. If FALSE, alpha is used in the second level of hypothesis testing. It is also effective when type="bic" (or 'aic') and check.dslope=TRUE, see Details.

If FALSE the final fit is returned silently with the selected number of breakpoints, otherwise the messages about the selection procedure (i.e. the BIC values), and possible warnings are also printed.

If TRUE the information criterion values with respect to the number of breakpoints are plotted. Ignored if type='score' or type='davies' or G>1. Note that if check.dslope=TRUE, the final number of breakpoints could differ from the one selected by the BIC/AIC leading to an inconsistent plot of the information criterion, see Note below.

When a large number of breakpoints is being tested, it could happen that 2 estimated breakpoints are too close each other, and only one can be retained. Thus if the difference between two breakpoints is less or equal to th, one (the first) breakpoint is removed. Of course, th depends on the x scale: Integers, like 5 or 10, are appropriate if the covariate is the observation index. Default (NULL) means th=diff(range(x))/100. Set th=0 if you are willing to consider even breakpoints very clode each other. Ignored if type='score' or type='davies'.

Number of sub-intervals to consider to search for the breakpoints when type='bic' or 'aic'. See Details.

Logical. Effective only if type='bic' or 'aic'. After the optimal number of breakpoints has been selected (via AIC/BIC), should the t values of the slope differences be checked? If TRUE, the breakpoints corresponding to slope differences with a 'low' t values will be removed. Note the model is re-fitted at each removal and a new check is performed. Simulation evidence suggests that such strategy leads to better results. See Details.

### Details

The function uses properly the functions segmented, pscore.test or davies.test to select the 'optimal' number of breakpoints  $\emptyset,1,\ldots,$ Kmax. If type='bic' or 'aic', the procedure stops if the last stop.if fits have increasing values of the information criterion. Moreover, a breakpoint is removed if too close to other, actually if the difference between two consecutive estimates is less then th. Finally, if check.dslope=TRUE, breakpoints whose corresponding slope difference estimate is 'small' (i.e. p-value larger then alpha or alpha/Kmax) are also removed.

When G>1 the dataset is split into G groups, and the search is carried out separately within each group. This approach is fruitful when there are many breakpoints not evenly spaced in the covariate range and/or concentrated in some sub-intervals. G=3 or 4 is recommended based on simulation evidence.

th

msg

plot.ic

bonferroni

G

check.dslope

Note Kmax is always tacitely reduced in order to have at least 1 residual df in the model with Kmax changepoints. Namely, if n=20, the maximal segmented model has 2\*(Kmax + 1) parameters, and therefore the largest Kmax allowed is 8.

When type='score' or 'davies', the function also returns the 'overall p-value' coming from combing the single p-values using the Fisher method. The pooled p-value, however, does not affect the final result which depends on the single p-values only.

#### Value

The returned object depends on argument return.fit. If FALSE, the returned object is a list with some information on the compared models (i.e. the BIC values), otherwise a classical 'segmented' object (see segmented for details) with the component selection.psi including the A/BIC values and

- if refit=TRUE, psi.no.refit that represents the breakpoint values before the last fit (with boot restarting)
- if G>1, cutvalues including the cutoffs values used to split the data.

#### Note

If check.dslope=TRUE, there is no guarantee that the final model has the lowest AIC/BIC. Namely the model with the best A/BIC could have 'non-significant' slope differences which will be removed (with the corresponding breakpoints) by the final model. Hence the possible plot (obtained via plot.ic=TRUE) could be misleading. See Example 1 below.

## Author(s)

Vito M. R. Muggeo

#### References

Muggeo V (2020) Selecting number of breakpoints in segmented regression: implementation in the R package segmented https://www.researchgate.net/publication/343737604

### See Also

```
segmented, pscore.test, davies.test
```

## **Examples**

```
set.seed(12)
xx<-1:100
zz<-runif(100)
yy<-2+1.5*pmax(xx-35,0)-1.5*pmax(xx-70,0)+15*pmax(zz-.5,0)+rnorm(100,0,2)
dati<-data.frame(x=xx,y=yy,z=zz)
out.lm<-lm(y~x,data=dati)
os <-selgmented(out.lm) #selection (Kmax=2) via the Score test (default)
os <-selgmented(out.lm, type="bic", Kmax=3) #BIC-based selection</pre>
```

slope

```
## Not run:
#Example 1: selecting a large number of breakpoints
b \leftarrow c(-1, rep(c(1.5, -1.5), l=15))
psi <- seq(.1,.9,1=15)
n <- 2000
x <- 1:n/n
X \leftarrow cbind(x, outer(x,psi,function(x,y)pmax(x-y,0)))
mu <- drop(tcrossprod(X,t(b)))</pre>
set.seed(113)
y < - mu + rnorm(n)*.022
par(mfrow=c(1,2))
#select number of breakpoints via the BIC (and plot it)
o<-selgmented(y, Kmax=20, type='bic', plot.ic=TRUE, check.dslope = FALSE)
plot(o, res=TRUE, col=2, lwd=3)
points(o)
# select via the BIC + check on the slope differences (default)
o1 <-selgmented(y, Kmax=20, type='bic', plot.ic=TRUE) #check.dslope = TRUE by default
#note the plot of BIC is misleading.. But the number of psi is correct
plot(o1, add=TRUE, col=3)
points(o1, col=3, pch=3)
#Example 2: a large number of breakpoints not evenly spaced.
b \leftarrow c(-1, rep(c(2, -2), l=10))
psi <- seq(.5,.9,l=10)
n <- 2000
x <- 1:n/n
X \leftarrow cbind(x, outer(x,psi,function(x,y)pmax(x-y,0)))
mu <- drop(tcrossprod(X,t(b)))</pre>
y < -mu + rnorm(n) *.02
#run selgmented with G>1. G=3 or 4 recommended.
#note G=1 does not return the right number of breaks
o1 <-selgmented(y, type="bic", Kmax=20, G=4)
## End(Not run)
```

slope

Slope estimates from segmented/stepmented relationships

## **Description**

Computes the slopes of each 'segmented' (or even 'stepmented') relationship in the fitted model.

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## Usage

```
slope(ogg, parm, conf.level = 0.95, rev.sgn=FALSE,
   APC=FALSE, .vcov=NULL, .coef=NULL, use.t=NULL, by=NULL,
   interc=TRUE, level=0, ..., digits = max(4, getOption("digits") - 2))
```

## **Arguments**

. 6	guments			
	ogg	an object of class "segmented" or "segmented.lme", returned by any segmented method or a list of two segmented fits to compare the estimates of corresponding slopes.		
	parm	the segmented variable whose slopes have to be computed. If missing all the segmented variables are considered.		
	conf.level	the confidence level required.		
	rev.sgn	vector of logicals. The length should be equal to the length of parm, but it is recycled otherwise. When TRUE it is assumed that the current parm is 'minus' the actual segmented variable, therefore the sign is reversed before printing. This is useful when a null-constraint has been set on the last slope.		
	APC	logical. If APC=TRUE the 'annual percent changes', i.e. $100 \times (\exp(\beta) - 1)$ , are computed for each interval ( $\beta$ is the slope). Only point estimates and confidence intervals are returned.		
	.vcov	The <i>full</i> covariance matrix of estimates. If unspecified (i.e. NULL), the covariance matrix is computed internally by vcov(ogg).		
	.coef	The regression parameter estimates. If unspecified (i.e. NULL), it is computed internally by $coef(ogg)$ .		
	use.t	Which quantiles should be used to compute the confidence intervals? If NULL (default) the $t$ distribution is used only for objects obtained by segmented.1m.		
	by	Only for segmented. Ime objects. It is a named list indicating covariate names and corresponding values affecting the fitted segmented relationship. For instance, by=list(group="2",z2=.2), provided that the model has been fitted by specifying group and z2 in x.diff (or as interaction with the segmented variable). Note that if the provided variables or values are irrelevant for changing the slopes, a warning message is printed.		
	interc	logical, only for 'stepmented' fits. If TRUE, the mean levels also account for the intercept; otherwise the first level is assumed to be zero.		
	level	Numeric, only for 'segmented.lme' fits. If $\emptyset$ , fixed effects left/right slopes are returned, otherwise the subject-specific values (with no confidence intervals).		
		Further arguments to be passed on to vcov.segmented, such as var.diff and is. See Details in vcov.segmented and summary.segmented.		
	digits	controls number of digits in the returned output.		

## **Details**

To fit broken-line relationships, segmented uses a parameterization whose coefficients are not the slopes. Therefore given an object "segmented", slope computes point estimates, standard errors, t-values and confidence intervals of the slopes of each segmented relationship in the fitted model.

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#### Value

slope returns a list of matrices. Each matrix represents a segmented relationship and its number of rows equal to the number of segments, while five columns summarize the results.

#### Note

The returned summary is based on limiting Gaussian distribution for the model parameters involved in the computations. Sometimes, even with large sample sizes such approximations are questionable (e.g., with small difference-in-slope parameters) and the results returned by slope might be unreliable. Therefore is responsability of the user to gauge the applicability of such asymptotic approximations. Anyway, the t values may be not assumed for testing purposes and they should be used just as guidelines to assess the estimate uncertainty.

### Author(s)

```
Vito M. R. Muggeo, <vito.muggeo@unipa.it>
```

#### References

Muggeo, V.M.R. (2003) Estimating regression models with unknown break-points. *Statistics in Medicine* **22**, 3055–3071.

#### See Also

See also davies. test and pscore. test to test for a nonzero difference-in-slope parameter.

## **Examples**

```
set.seed(16)
x<-1:100
y<-2+1.5*pmax(x-35,0)-1.5*pmax(x-70,0)+rnorm(100,0,3)
out < -glm(y^1)
out.seg<-segmented(out,seg.Z=x,psi=list(x=c(20,80)))
## the slopes of the three segments....
slope(out.seg)
rm(x,y,out,out.seg)
## an heteroscedastic example..
set.seed(123)
n<-100
x<-1:n/n
y < -x+1.5*pmax(x-.5,0)+rnorm(n,0,1)*ifelse(x < -.5,.4,.1)
oseg<-segmented(o,seg.Z=~x,psi=.6)
slope(oseg)
slope(oseg,var.diff=TRUE) #better CI
```

stagnant 69

stagnant

Stagnant band height data

## **Description**

The stagnant data frame has 28 rows and 2 columns.

## Usage

```
data(stagnant)
```

#### **Format**

A data frame with 28 observations on the following 2 variables.

- x log of flow rate in g/cm sec.
- y log of band height in cm

## **Details**

Bacon and Watts report that such data were obtained by R.A. Cook during his investigation of the behaviour of stagnant surface layer height in a controlled flow of water.

### **Source**

Bacon D.W., Watts D.G. (1971) Estimating the transistion between two intersecting straight lines. *Biometrika* **58**: 525 – 534.

Originally from the PhD thesis by R.A. Cook

# **Examples**

```
data(stagnant)
## plot(stagnant)
```

step.lm.fit

Fitter Functions for stepmented Linear Models

# Description

step.lm.fit is called by stepmented.lm to fit stepmented linear (gaussian) models. Likewise, step.glm.fit is called by stepmented.glm to fit generalized stepmented linear models. The step.\*.fit.boot functions are employed to perform bootstrap restarting. These functions should usually not be used directly by the user.

step.lm.fit

### Usage

## **Arguments**

У	vector of observations of length n.
x.lin,XREG	design matrix for standard linear terms.
Xtrue, Z	appropriate matrix including the stepmented variables whose breakpoints have to be estimated.
PSI	appropriate matrix including the starting values of the breakpoints to be estimated.
ww, w	possibe weights vector.
offs	possibe offset vector.
opz	a list including information useful for model fitting.
n.boot	the number of bootstrap samples employed in the bootstrap restart algorithm.
break.boot	Integer, less than n.boot. If break.boot consecutive bootstrap samples lead to the same objective function, the algorithm stops without performing all n.boot 'trials'. This can save computational time considerably.
size.boot	the size of the bootstrap resamples. If NULL (default), it is taken equal to the sample size. values smaller than the sample size are expected to increase perturbation in the bootstrap resamples.
jt	logical. If TRUE the values of the stepmented variable(s) are jittered before fitting the model to the bootstrap resamples.
nonParam	if TRUE nonparametric bootstrap (i.e. case-resampling) is used, otherwise residual-based.
random	if TRUE, when the algorithm fails to obtain a solution, random values are used as candidate values.
return.all.sol	if TRUE, when the algorithm fails to obtain a solution, the values visited by the algorithm with corresponding deviances are returned.

## **Details**

The functions call iteratively lm.wfit (or glm.fit) with proper design matrix depending on XREG, Z and PSI. step.lm.fit.boot (and step.glm.fit.boot) implements the bootstrap restarting idea discussed in Wood (2001).

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### Value

A list of fit information.

#### Note

These functions should usually not be used directly by the user.

### Author(s)

Vito Muggeo

#### References

Wood, S. N. (2001) Minimizing model fitting objectives that contain spurious local minima by bootstrap restarting. *Biometrics* **57**, 240–244.

### See Also

```
stepmented.lm or stepmented.glm
```

### **Examples**

##See ?stepmented

stepmented

stepmented relationships in regression models

### **Description**

Fits regression models with stepmented (i.e. piecewise-constant) relationships between the response and one or more explanatory variables. Break-point estimates are provided.

## Usage

```
stepmented(obj, seg.Z, psi, npsi, fixed.psi=NULL, control = seg.control(),
   keep.class=FALSE, var.psi=FALSE, ...)
## S3 method for class 'lm'
stepmented(obj, seg.Z, psi, npsi, fixed.psi=NULL, control = seg.control(),
   keep.class=FALSE, var.psi=FALSE, ...)
## S3 method for class 'glm'
stepmented(obj, seg.Z, psi, npsi, fixed.psi=NULL, control = seg.control(),
   keep.class=FALSE, var.psi=FALSE, ...)
## S3 method for class 'numeric'
stepmented(obj, seg.Z, psi, npsi, fixed.psi=NULL, control = seg.control(),
   keep.class=FALSE, var.psi=FALSE, ...,
   pertV=0, centerX=FALSE, adjX=NULL, weights=NULL)
```

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```
## S3 method for class 'ts'
stepmented(obj, seg.Z, psi, npsi, fixed.psi=NULL, control = seg.control(),
   keep.class=FALSE, var.psi=FALSE, ...,
   pertV=0, centerX=FALSE, adjX=NULL)
```

### **Arguments**

seg.Z

obj A standard 'linear' regression model of class "lm" or "glm". Alternatively, a simple "ts" object or a simple data vector may be supplied.

the stepmented variables(s), i.e. the numeric covariate(s) understood to have a piecewise-constant relationship with response. It is a formula with no response variable, such as seg.Z=~x or seg.Z=~x1+x2. Currently, formulas involving functions, such as seg.Z=~log(x1), or selection operators, such as seg.Z=~d[,"x1"] or seg.Z=~d\$x1, are *not* allowed. Also, variable names formed by U or V only (with or without numbers) are not permitted. If missing, the index variable id=1,2,..,n is used. For stepmented.ts, seg.Z is usually unspecified as the (time) covariate is obtained by the ts object itself.

starting values for the breakpoints to be estimated. If there is a single stepmented variable specified in seg. Z, psi can be a numeric vector, and it can be missing when 1 breakpoint has to be estimated (and the median of the stepmented variable is used as a starting value). If seg. Z includes several covariates, psi has to be specified as a *named* list of vectors whose names have to match the variables in the seg. Z argument. Each vector of such list includes starting values for the break-point(s) for the corresponding variable in seg. Z. A NA value means that 'K' quantiles (or equally spaced values) are used as starting values; K is fixed via the seg.control auxiliary function.

A named vector or list meaning the *number* (and not locations) of breakpoints to be estimated. The starting values will be internally computed via the quantiles or equally spaced values, as specified in argument quant in seg.control. npsi can be missing and npsi=1 is assumed for all variables specified in seg.Z. If psi is provided, npsi is ignored.

An optional named list including the breakpoint values to be kept fixed during the estimation procedure. The names should be a subset of (or even the same) variables specified in seg.Z. If there is a single variable in seg.Z, a simple numeric vector can be specified. Note that, in addition to the values specified here, stepmented will estimate additional breakpoints. To keep fixed all breakpoints (to be specified in psi) use it.max=0 in seg.control

a list of parameters for controlling the fitting process. See the documentation for seg.control for details.

logical value indicating if the final fit returned by stepmented.default should keep the class 'stepmented' (along with the class of the original fit obj). Ignored by the stepmented methods.

optional arguments (to be ignored safely). Notice specific arguments relevant to the original call (via lm or glm for instance), such as weights or offet, have to

psi

npsi

fixed.psi

control

keep.class

• • •

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be included in the starting model obj.

pertV Only for stepmented.ts and stepmented.numeric.

centerX Only for stepmented.ts and stepmented.numeric. If TRUE, the covariate is

centered before fitting.

adjX Only for stepmented.ts and stepmented.numeric. If the response vector

leads to covariate with large values (such as years for ts objects), adjX=TRUE will shift the covariate to have a zero origin. Default is NULL which means TRUE

if the minimum of covariate is 1000 or larger.

var.psi logical. If TRUE, the estimate covariance matrix is also computed via vcov. stepmented,

thus the breakpoint standard errors are also included in the psi component of the returned object. Default is FALSE, as computing the estimate covariance matrix

is somewhat time-consuming when the sample size is large.

weights possible weights to include in the estimation process (only for stepmented.numeric).

#### **Details**

Given a linear regression model (usually of class "lm" or "glm"), stepmented tries to estimate a new regression model having piecewise-constant (i.e. step-function like) relationships with the variables specified in seg. Z. A *stepmented* relationship is defined by the mean level parameters and the breakpoints where the mean level changes. The number of breakpoints of each stepmented relationship depends on the psi argument, where initial values for the break-points must be specified. The model is estimated simultaneously yielding point estimates and relevant approximate standard errors of all the model parameters, including the break-points.

stepmented implements the algorithm described in Fasola et al. (2018) along with bootstrap restarting (Wood, 2001) to escape local optima. The procedure turns out to be particularly appealing and probably efficient when there are two or more covariates exhibiting different change points to be estimated.

#### Value

The returned object is of class "stepmented" which inherits from the class "lm" or "glm" depending on the class of obj. When only.mean=FALSE, it is a list having two 'stepmented' fits (for the mean and for the dispersion submodels).

An object of class "stepmented" is a list containing the components of the original object obj with additionally the followings:

psi estimated break-points and relevant (approximate) standard errors (on the con-

tinuum)

psi.rounded the rounded estimated break-points (see Note, below)

it number of iterations employed

epsilon difference in the objective function when the algorithm stops

model the model frame

psi.history a list or a vector including the breakpoint estimates at each step

seed the integer vector containing the seed just before the bootstrap resampling. Re-

turned only if bootstrap restart is employed

. . Other components are not of direct interest of the user

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#### Note

The component psi.rounded of the fit object includes the rounded changepoint values which are usually taken as the final estimates. More specifically, each column of psi.rounded represents a changepoint and the corresponding rows are the range of the 'optimal' interval. The first row, i.e. the lower bound of the interval, is taken as point estimate. print.stepmented, print.summary.stepmented, and confint.stepmented return the rounded (lower) value of the interval.

#### Also:

1. The algorithm will start if the it.max argument returned by seg.control is greater than zero. If it.max=0 stepmented will estimate a new linear model with break-point(s) fixed at the starting values reported in psi. Alternatively, it is also possible to set h=0 in seg.control(). In this case, bootstrap restarting is unnecessary, then to have changepoints at mypsi type

```
stepmented(.., psi=mypsi, control=seg.control(h=0, n.boot=0, it.max=1))
```

- 2. In the returned fit object, 'U.' is put before the name of the stepmented variable to indicate the difference in the mean levels. slope can be used to compute the actual mean levels corresponding to the different intervals.
- 3. Currently methods specific to the class "stepmented" are
  - print.stepmented
  - summary.stepmented
  - print.summary.stepmented
  - plot.stepmented
  - confint.stepmented
  - vcov.stepmented
  - lines.stepmented

Others are inherited from the class "lm" or "glm" depending on the class of obj.

#### Author(s)

Vito M. R. Muggeo, <vito.muggeo@unipa.it> (based on original code by Salvatore Fasola)

#### References

Fasola S, Muggeo VMR, Kuchenhoff H (2018) A heuristic, iterative algorithm for change-point detection in abrupt change models, *Computational Statistics* **33**, 997–1015

#### See Also

segmented for segmented regression, lm, glm

## **Examples**

```
n=20
x<-1:n/n
mu<- 2+ 1*(x>.6)
y<- mu + rnorm(n)*.8
```

summary.segmented 75

```
#fitting via regression model
os <-stepmented(lm(y^1), x)
y<-ts(y)
os1<- stepmented(y) #the 'ts' method
os2<- stepmented(y, npsi=2)
#plot(y)
#plot(os1, add=TRUE)
#plot(os2, add=TRUE, col=3:5)
### Example with (poisson) GLM
y<- rpois(n,exp(mu))</pre>
o<-stepmented(glm(y^1,family=poisson))
plot(o, res=TRUE)
## Not run:
## Example using the (well-known) Nile dataset
data(Nile)
plot(Nile)
os<- stepmented(Nile)</pre>
plot(os, add=TRUE)
### Example with (binary) GLM (example from the package stepR)
set.seed(1234)
y \leftarrow rbinom(200, 1, rep(c(0.1, 0.7, 0.3, 0.9), each=50))
o<-stepmented(glm(y~1,family=binomial), npsi=3)
plot(o, res=TRUE)
### Two stepmented covariates (with 1 and 2 breakpoints); z has also an additional linear effect
n=100
x<-1:n/n
z < -runif(n, 2, 5)
mu<-2+1*(x>.6)-2*(z>3)+3*(z>4)+z
y < - mu + rnorm(n)*.8
os <-stepmented(lm(y^z), ^x+z, npsi=c(x=1,z=2))
os
summary(os)
## see ?plot.stepmented
## End(Not run)
```

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## **Description**

summary method for class segmented.

#### Usage

```
## S3 method for class 'segmented'
summary(object, short = FALSE, var.diff = FALSE, p.df="p", .vcov=NULL, ...)
## S3 method for class 'summary.segmented'
print(x, short=x$short, var.diff=x$var.diff,
    digits = max(3, getOption("digits") - 3),
    signif.stars = getOption("show.signif.stars"),...)
```

# **Arguments**

object	Object of class "segmented".
short	logical indicating if the 'short' summary should be printed.
var.diff	logical indicating if different error variances should be computed in each interval of the segmented variable, see Details. If .vcov is provided, var.diff is set to FALSE.
p.df	A character as a function of 'p' (number of parameters) and 'K' (number of groups or segments) affecting computations of the group-specific variance (and the standard errors) if var.diff=TRUE, see Details.
. VCOV	Optional. The full covariance matrix for the parameter estimates. If provided, standard errors are computed (and displayed) according to this matrix.
X	a summary.segmented object produced by summary.segmented().
digits	controls number of digits printed in output.
signif.stars	logical, should stars be printed on summary tables of coefficients?
	further arguments.

### **Details**

If short=TRUE only coefficients of the segmented relationships are printed. If var.diff=TRUE and there is only one segmented variable, different error variances are computed in the intervals defined by the estimated breakpoints of the segmented variable. For the jth interval with  $n_j$  observations, the error variance is estimated via  $RSS_j/(n_j-p)$ , where  $RSS_j$  is the residual sum of squares in interval j, and p is the number of model parameters. This number to be subtracted from  $n_j$  can be changed via argument p.df. For instance p.df="0" uses  $RSS_j/(n_j)$ , and p.df="p/K" leads to  $RSS_j/(n_j-p/K)$ , where K is the number of groups (segments), and p/K can be interpreted as the average number of model parameter in that group.

Note var.diff=TRUE only affects the estimates covariance matrix. It does *not* affect the parameter estimates, neither the log likelihood and relevant measures, such as AIC or BIC. In other words, var.diff=TRUE just provides 'alternative' standard errors, probably appropriate when the error variances are different before/after the estimated breakpoints. Also p-values are computed using the t-distribution with 'naive' degrees of freedom (as reported in object\$df.residual).

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If var.diff=TRUE the variance-covariance matrix of the estimates is computed via the sandwich formula,

$$(X^T X)^{-1} X^T V X (X^T X)^{-1}$$

where V is the diagonal matrix including the different group-specific error variance estimates. Standard errors are the square root of the main diagonal of this matrix.

#### Value

A list (similar to one returned by segmented.lm or segmented.glm) with additional components:

psi	estimated break-points and relevant (approximate) standard errors
Ttable	estimates and standard errors of the model parameters. This is similar to the matrix coefficients returned by summary.lm or summary.glm, but without the rows corresponding to the breakpoints. Even the p-values relevant to the difference-in-slope parameters have been replaced by NA, since they are meaningless in this case, see davies.test.
gap	estimated coefficients, standard errors and t-values for the 'gap' variables
cov.var.diff	if var.diff=TRUE, the covaraince matrix accounting for heteroscedastic errors.
sigma.new	if var.diff=TRUE, the square root of the estimated error variances in each interval.
df.new	if var.diff=TRUE, the residual degrees of freedom in each interval.

# Author(s)

Vito M.R. Muggeo

## See Also

```
print.segmented, davies.test
```

# **Examples**

```
##continues example from segmented()
# summary(segmented.model,short=TRUE)

## an heteroscedastic example..
# set.seed(123)
# n<-100
# x<-1:n/n
# y<- -x+1.5*pmax(x-.5,0)+rnorm(n,0,1)*ifelse(x<=.5,.4,.1)
# o<-lm(y~x)
# oseg<-segmented(o,seg.Z=~x,psi=.6)
# summary(oseg,var.diff=TRUE)$sigma.new</pre>
```

summary.segmented.lme Summarizing model fits for segmented mixed-effects regression

# **Description**

summary method for class segmented.lme.

#### Usage

```
## S3 method for class 'segmented.lme'
summary(object, .vcov=NULL, digits = max(3, getOption("digits") - 3), ...)
```

# **Arguments**

object	Object of class "segmented.lme".
.vcov	Optional. The full covariance matrix for the parameter estimates. If provided, standard errors are computed (and displayed) according to this matrix.
digits	controls number of digits printed in output.
	further arguments.

# **Details**

The function summarizes and prints the most relevant information on the segmented mixed fit. The output is similar to that returned by print.summary.lme

#### Value

A list (similar to one returned by segmented.lm) with estimates of the variance components, and point estimates, standard errors, DF, t-value and p-value for the fixed effects. p-values for the variables U and G0 are omitted as pointless.

#### Author(s)

```
Vito M.R. Muggeo
```

## See Also

```
print.segmented.lme
```

#### **Examples**

```
##continues example from segmented.lme()
# summary(os)
```

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summary.stepmented

Summarizing model fits for stepmented regression

#### Description

summary/print method for class stepmented.

## Usage

```
## S3 method for class 'stepmented'
summary(object, short = FALSE, var.diff = FALSE, p.df="p", .vcov=NULL, ...)
## S3 method for class 'summary.stepmented'
print(x, short=x$short, var.diff=x$var.diff,
    digits = max(3, getOption("digits") - 3),
    signif.stars = getOption("show.signif.stars"),...)
## S3 method for class 'stepmented'
print(x, digits = max(3, getOption("digits") - 3),
    ...)
```

#### **Arguments**

object, x	Object of class "stepmented" or a summary.stepmented object produced by summary.stepmented().
short	logical indicating if the 'short' summary should be printed.
var.diff	logical indicating if different error variances should be computed in each interval of the stepmented variable, see Details. If .vcov is provided, var.diff is set to FALSE.
p.df	A character as a function of 'p' (number of parameters) and 'K' (number of groups or segments) affecting computations of the group-specific variance (and the standard errors) if var.diff=TRUE, see Details.
. VCOV	Optional. The full covariance matrix for the parameter estimates. If provided, standard errors are computed (and displayed) according to this matrix.
digits	controls number of digits printed in output.
signif.stars	logical, should stars be printed on summary tables of coefficients?
•••	further arguments, notably type to be passed to vcov.stepmented to compute the standard errors. See vcov.stepmented.

#### **Details**

If short=TRUE only coefficients of the stepmented relationships are printed. If var.diff=TRUE and there is only one stepmented variable, different error variances are computed in the intervals defined by the estimated breakpoints of the stepmented variable. For the jth interval with  $n_j$  observations, the error variance is estimated via  $RSS_j/(n_j-p)$ , where  $RSS_j$  is the residual sum of squares in

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interval j, and p is the number of model parameters. This number to be subtracted from  $n_j$  can be changed via argument p.df. For instance p.df="0" uses  $RSS_j/(n_j)$ , and p.df="p/K" leads to  $RSS_j/(n_j-p/K)$ , where K is the number of groups (segments), and p/K can be interpreted as the average number of model parameter in that group.

Note var.diff=TRUE only affects the estimates covariance matrix. It does *not* affect the parameter estimates, neither the log likelihood and relevant measures, such as AIC or BIC. In other words, var.diff=TRUE just provides 'alternative' standard errors, probably appropriate when the error variances are different before/after the estimated breakpoints. Also p-values are computed using the t-distribution with 'naive' degrees of freedom (as reported in object\$df.residual).

If var.diff=TRUE the variance-covariance matrix of the estimates is computed via the sandwich formula,

$$(X^T X)^{-1} X^T V X (X^T X)^{-1}$$

where V is the diagonal matrix including the different group-specific error variance estimates. Standard errors are the square root of the main diagonal of this matrix.

#### Value

A list (similar to one returned by stepmented.lm or stepmented.glm) with additional components:

psi	estimated break-points and relevant (approximate) standard errors
Ttable	estimates and standard errors of the model parameters. This is similar to the matrix coefficients returned by summary.lm or summary.glm, but without the rows corresponding to the breakpoints. Even the p-values relevant to the difference-in-slope parameters have been replaced by NA, since they are meaningless in this case, see davies.test.
cov.var.diff	if var.diff=TRUE, the covaraince matrix accounting for heteroscedastic errors.
sigma.new	if var.diff=TRUE, the square root of the estimated error variances in each interval.
df.new	if var.diff=TRUE, the residual degrees of freedom in each interval.

# Warning

If type is not specified in . . . (which means type="standard"), no standard error will be computed (and returned) for the jumpoint.

#### Author(s)

Vito M.R. Muggeo

#### See Also

pscore.test

vcov.segmented 81

#### **Examples**

```
##continues example from stepmented()
# summary(stepmented.model,short=TRUE)

## an heteroscedastic example..
# set.seed(123)
# n<-100
# x<-1:n/n
# y<- -x+1.5*pmax(x-.5,0)+rnorm(n,0,1)*ifelse(x<=.5,.4,.1)
# o<-lm(y~x)
# oseg<-stepmented(o,seg.Z=~x,psi=.6)
# summary(oseg,var.diff=TRUE)$sigma.new</pre>
```

vcov.segmented

Variance-Covariance Matrix for a Fitted Segmented Model

#### **Description**

Returns the variance-covariance matrix of the parameters (including breakpoints) of a fitted segmented model object.

#### Usage

```
## S3 method for class 'segmented'
vcov(object, var.diff = FALSE, is = FALSE, ...)
```

## **Arguments**

object	a fitted model object of class "segmented", returned by any segmented method or segreg.
var.diff	logical. If var.diff=TRUE and there is a single segmented variable, the covariance matrix is computed using a sandwich-type formula. See Details in summary.segmented.
is	logical. If TRUE, the <i>asymptotic</i> covariance matrix based on the idea of induced smoothing is returned. If $is=TRUE$ , $var.diff=FALSE$ is set. $is=TRUE$ only works with segmented (g)lm fits.
	additional arguments.

## **Details**

The returned covariance matrix is based on an approximation of the nonlinear segmented term. Therefore covariances corresponding to breakpoints are reliable only in large samples and/or clear cut segmented relationships. If is=TRUE, the returned covariance matrix depends on the design matrix having the term  $I(x>\psi)$  replaced by its smooth counterpart.

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## Value

The full matrix of the estimated covariances between the parameter estimates, including the breakpoints.

## Note

var.diff=TRUE works when there is a single segmented variable.

## Author(s)

```
Vito M. R. Muggeo, <vito.muggeo@unipa.it>
```

## See Also

```
summary.segmented
```

# **Examples**

```
##continues example from summary.segmented()
# vcov(oseg)
# vcov(oseg, var.diff=TRUE)
# vcov(oseg, is=TRUE)
```

vcov.segmented.lme

Variance-Covariance Matrix for a Fitted Segmented Mixed Model

# Description

Returns the variance-covariance matrix of the parameters (including breakpoints) of a fitted segmented mixed model object.

# Usage

```
## S3 method for class 'segmented.lme'
vcov(object, B=0, ret.b=FALSE, ...)
```

## **Arguments**

object	a fitted model object of class "segmented.lme", returned by segmented.lme method.
В	number of bootstrap replicates, if a bootstrap-based covariance matrix is requested.
ret.b	logical. If FALSE the full covariance matrix (for the fixed effect estimates) based on B case-resampling bootstrap samples is returned; otherwise a list with information on the bootstrap sampling distributions.
• • •	optional arguments, i.e. seed and it.max.b, used when implementing the bootstrap.

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#### **Details**

The returned covariance matrix is based on an approximation of the nonlinear segmented term. Therefore covariances corresponding to breakpoints are reliable only in large samples and/or clear cut segmented relationships. If B>0 is set, case resampling bootstrap (on the outermost nesting level) is carried out. Moreover, if ret.b=TRUE, the bootstrap distributions are returned, rather than the covariance matrix.

#### Value

The full matrix of the estimated covariances of the fixed effects estimates, including the breakpoint.

## Warning

All the functions for segmented mixed models (\*.segmented.lme) are still at an experimental stage

#### Author(s)

```
Vito M. R. Muggeo, <vito.muggeo@unipa.it>
```

#### See Also

```
summary.segmented.lme
```

#### **Examples**

```
##continues example from segmented.lme()
# vcov(os)
# vcov(os, B=50)
# vcov(os, B=50, ret.b=TRUE)
```

vcov.stepmented

Variance-Covariance Matrix for a Fitted Stepmented Model

# Description

Returns the variance-covariance matrix of the parameters estimates (including breakpoints) of a fitted stepmented model object.

# Usage

```
## S3 method for class 'stepmented'
vcov(object, k=NULL, zero.cor=TRUE, type=c("cdf", "none", "abs"), ...)
```

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#### **Arguments**

object a fitted model object of class "stepmented", returned by any stepmented method k The power of n for the smooth approximation. Simulation evidence suggests k in [-1, -1/2]; with k = -1/2 providing somewhat 'conservative' standard errors especially at small sample sizes. In general, the larger k, the smaller  $n^{-k}$ , and the smaller the jumpoint standard error. If TRUE, the covariances between the jumpoints and the remaining linear coeffizero.cor cients are set to zero (as theory states). How the covariance matrix should be computed. If "none", the usual asymptotic type covariance matrix for the linear coefficients only (under homoskedasticity and assuming known the jumpoints) is returned; if "cdf", the standard normal cdf is used to approximate the indicator function (see details); "abs" is yet another approximation (currently unimplemented). additional arguments.

#### **Details**

The full covariance matrix is based on the smooth approximation

$$I(x > \psi) \approx \Phi((x - \psi)/n^k)$$

via the sandwich formula using the empirical information matrix and assuming  $x \in [0, 1]$ .  $\Phi(\cdot)$  is the standard Normal cdf, and k is the argument k. When k=NULL (default), it is computed via

$$k = -(0.6 + 0.5 \log(snr)/\sqrt{snr} - (|\hat{\psi} - 0.5|/n)^{1/2})$$

where snr is the signal-to-noise ratio corresponding to the estimated changepoint  $\hat{\psi}$  (in the range (0,1)). The above formula comes from extensive simulation studies under different scenarios: Seo and Linton (2007) discuss using the normal cdf to smooth out the indicator function by suggesting  $\log(n)/n^{1/2}$  as bandwidth; we found such suggestion does not perform well in practice.

#### Value

The full matrix of the estimated covariances between the parameter estimates, including the breakpoints.

## Warning

The function, including the value of k, must be considered at preliminary stage. Currently the value of k appears to overestimate slightly the true  $\hat{\psi}$  variability.

#### Note

If the fit object has been called by stepmented(.., var.psi=TRUE), then vcov.stepmented will return object\$vcov, unless the power k differs from -2/3.

### Author(s)

Vito Muggeo

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# References

Seo MH, Linton O (2007) A smoothed least squares estimator for threshold regression models, J of Econometrics, 141:704-735

# See Also

stepmented

# Examples

##see ?stepmented

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