Real-time Recession Probability with Hidden Markov Model and Unemployment Momentum

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Abstract We show how to construct a composite Hidden Markov Model (HMM) to calculate real-time recession probability, using the jubilee and ldhmm packages in R. The input data is the unemployment rate (UNRATE) which is released monthly by the U.S. government. There are two sub-models: The one-year momentum model and the 6-month acceleration model. The product of the two generates the recession probability. Our model demonstrates that positive momentum in unemployment kicks off a recession. The momentum accelerates during the recession. And eventually the rapid deceleration marks the end of it.

Introduction

In Section 9 of Lihn (2019), we developed a composite Hidden Markov Model (HMM) to calculate real-time recession probability, using the jubilee and ldhmm packages in R (R Core Team (2019)). The input data is the unemployment rate (UNRATE) which is released monthly by the U.S. government (US BLS (2019)). The HMM in the ldhmm package utilizes mixtures of λ distribution to capture the kurtosis more accurately (Lihn (2017)) [1] , which would not be possible with normal mixtures. In this paper, we provide the technical detail about this recession probability model.

There are two sub-models: The one-year momentum model and the 6-month acceleration model. In the momentum sub-model, define the **unemployment momentum** as one-year log-return of UNRATE:

$$U_{1y}(t) \equiv \log UNRATE(t) - \log UNRATE(t-1). \tag{2}$$

$U_{1y}(t)$ is treated as a two-state HMM: The first state is the crash state, or called "the crash regime." And the second state is the normal state. The economy spends most of its time in the normal state, which is associated with negative momentum. That is, the unemployment rate is decreasing, and the job market is improving. On the other hand, the crash state is associated with large positive momentum. In this state, the economy loses jobs in a rapid pace.

The two regime scenario is consistent with the regime-switching findings in Bae Kim, Mulvey (2014) and Mulvey and Liu (2016). On the other hand, when $U_{1y}(t) > 0$, we note that $U_{1y}(t)$ is approximately 1/6 of Sahm's unemployment index in Sahm (2019). The reason was explained in Section 9.1 of Lihn (2019).

For the acceleration sub-model, define the **unemployment acceleration** as the 6-month rate of change of $U_{1y}(t)$ such as

$$A_{6m}(t) \equiv 2\left(U_{1y}(t) - U_{1y}(t-0.5)\right). \tag{3}$$

$A_{6m}(t)$ is treated as a three-state HMM: the accelerating state, middle state, and decelerating state. Recession is often associated with the accelerating state.

Finally, the recession probability is the product of the probability in the crash state and the probability in the accelerating state. Our model demonstrates that positive momentum in unemployment kicks off a recession. The momentum accelerates during the recession. And the rapid decceleration marks the end of it.

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[1]The PDF of a λ distribution is a two-sided stretched exponential function, defined by the parameter tuple (μ, σ, λ),

$$P(x; \mu, \sigma, \lambda) = \frac{1}{\sigma \lambda \Gamma\left(\frac{\lambda}{2}\right)} e^{-\left|\frac{x-\mu}{\sigma}\right|^{\frac{2}{\lambda}}}. \tag{1}$$

The shape parameter $\lambda > 0$ is called the "order" of the distribution. The kurtosis is increasing with λ. When $\lambda = 1$, it converges to a normal distribution. In the context of stable distribution, we have the stability index $\alpha = 2/\lambda$.

Loading Package and Preparing Data

This paper is written in the reproducible research style. If the reader follows each command, he/she should obtain the same result. Very small difference may come from libraries using random numbers, such as the fitting of the normal mixtures.

We begin with loading the `jubilee` package and setting up several essential data tables:

```
> library(jubilee)
>
> set.seed(804)
> repo <- jubilee.repo(online=FALSE)

[1] "Maximum date in raw ie.data is 2019.12 and SPX average at 3223.38"
[1] "Maximum date for unrate is 2019-12-16 and for GDP, 2019-08-16"

> ju <- jubilee(repo@ie, lookback.channel=45, fwd.rtn.duration=10)
> dt <- ju@dtb
> dj <- ju@reg.dtb
```

The attributes that we are interested in this paper are:

- t = `dj$fraction` : Time in years. Each month is in the 1/12 unit. Note that we follow Shiller's "middle of the month" convention since he averages the quantity. But when we download the monthly data from FRED, we use the monthly data as is. There is no average involved in monthly data.
- $UNRATE\,(t)$ = `dj$unrate`: The unemployment rate.
- $U_{1y}\,(t)$ = `dj$unrate.logr.1`: The unemployment momentum $U_{1y}\,(t)$, which will be renamed to `unrate.mom` shortly.
- $A_{6m}\,(t)$ = `dj$unrate.logr.1.6m`: The unemployment acceleration $A_{6m}\,(t)$, which will be renamed to `unrate.acc` shortly.

We would like to limit our analysis to the training period of $t > 1956$. The data was too volatile prior to this time, which tends to distort the training of the HMM states.

We create the following target data table `rec.dtb` that contains all the macro data used in this paper. This is shown below:

```
> # J defines the training period, avoid all NA situations
> J <- which(dj$fraction > 1956 & is.finite(dj$unrate)
+            & is.finite(dj$unrate.logr.1) & is.finite(dj$unrate.logr.1.6m))
> rec.dtb <- data.table(
+     fraction = dj[J]$fraction,
+     unrate = dj[J]$unrate,
+     unrate.mom = dj[J]$unrate.logr.1,
+     unrate.acc = dj[J]$unrate.logr.1.6m,
+     usrec.nber = dj[J]$usrec.nber,
+     usrec.cp = dj[J]$usrec.cp
+ )
```

$U_{1y}\,(t)$ is now called `unrate.mom`, and $A_{6m}\,(t)$ is called `unrate.acc` for ease of memory. The `usrec.nber` column stores the U.S. recession binary probability from NBER (2019). The `usrec.cp` column stores the recession probability from Piger and Chauvet (2019).

Bootstrapping With Normal Mixtures

In order to use the `ldhmm` package, we must provide initial estimate of the HMM states. This can be accomplished by the method of normal mixtures. We call this "bootstrapping." We use the `mixtools` package in R as shown below:

```
> library(mixtools)
>
> rec <- list(
+     md_mom = mixtools::normalmixEM(rec.dtb$unrate.mom, k=2),
+     md_acc = mixtools::normalmixEM(rec.dtb$unrate.acc, k=3)
+ )
```

```
number of iterations= 51
number of iterations= 271
```

The momentum data is decomposed into two states (`lambda` is the state density, `mu` is the mean, and `sigma` the SD of the normal distribution):

```
> summary(rec$md_mom)

summary of normalmixEM object:
           comp 1   comp 2
lambda  0.6084015 0.391598
mu     -0.0815772 0.119816
sigma   0.0644040 0.212434
loglik at estimate:  415.9232
```

One state has positive mean, and the other has negative mean. This meets our expectation.

The acceleration data is decomposed into three states:

```
> summary(rec$md_acc)

summary of normalmixEM object:
          comp 1     comp 2   comp 3
lambda  0.243293 0.65197720 0.104730
mu     -0.157921 0.00276007 0.386037
sigma   0.450597 0.14032780 0.129627
loglik at estimate:  -72.87702
```

One state has negative mean. The second state has nearly zero mean, while the third state has positive mean. Notice that the negative-mean state has very large SD.

We plot the time series data and the normal mixture distributions in Figure 1.

HMM States for the Momentum Model

We introduce the following helper function `ldhmm.md2mle()`, which takes a normal mixture object `md` from the `mixtools` package as input, and performs MLE optimization to obtain the λ distribution states in the `ldhmm` package. The function also performs decoding and calculates the state probabilities.

```
> library(ldhmm)
>
> ldhmm.md2mle <- function(md) {
+         stopifnot(class(md) == "mixEM")
+     m <- length(md$mu)
+     ord <- rev(order(md$mu)) # large-mu state goes first
+     param0 <- t(rbind(md$mu[ord], md$sigma[ord], md$mu*0 + 1))
+     gamma0 <- ldhmm.gamma_init(m=m)
+     h <- ldhmm(m, param0, gamma0, NULL, stationary=TRUE)
+     x <- md$x
+     x[is.na(x)] <- 0 # just to be safe
+     hd <- ldhmm.mle(h, x, decode=TRUE, print.level=0)
+     rs <- list(
+         x = x,
+         md = md,
+         input = h,
+         output = hd,
+         prob = hd@states.prob[1,],
+         data_stats = hd@states.local.stats,
+         ld_stats = ldhmm.ld_stats(hd)
+     )
+     return(rs)
+ }
```

First, we train the momentum model. The following listing shows the statistics and parameters of the momentum states from the `ldhmm` package:

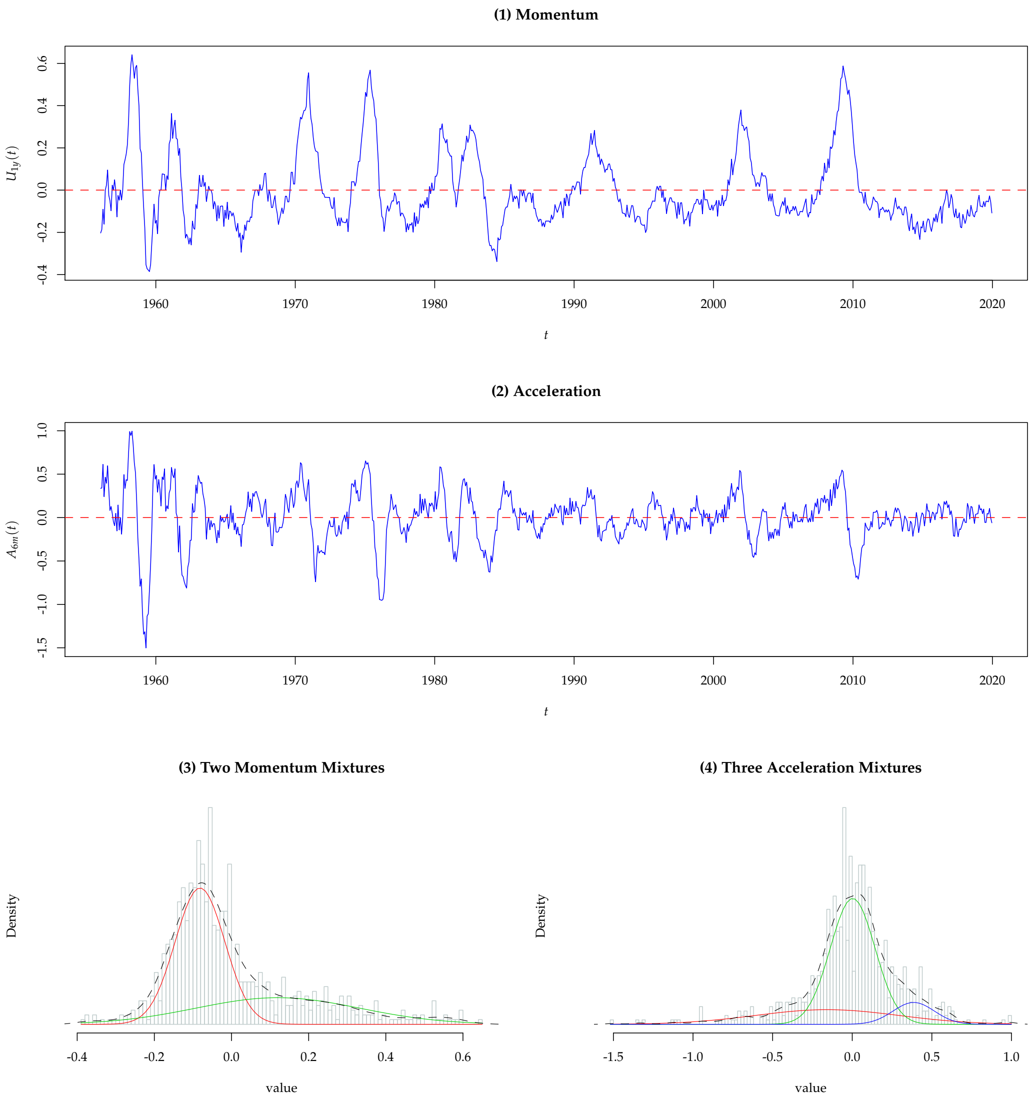

Figure 1: Momentum, Acceleration, and their normal mixtures. Panel (1) shows $U_{1y}(t)$ in the blue line. Panel (2) shows $A_{6m}(t)$ in the blue line. Panel (3) shows the two normal-mixture components (green and red) of the $U_{1y}(t)$ distribution. Panel (4) shows the three normal-mixture components (green, red, and blue) of the $A_{6m}(t)$ distribution.

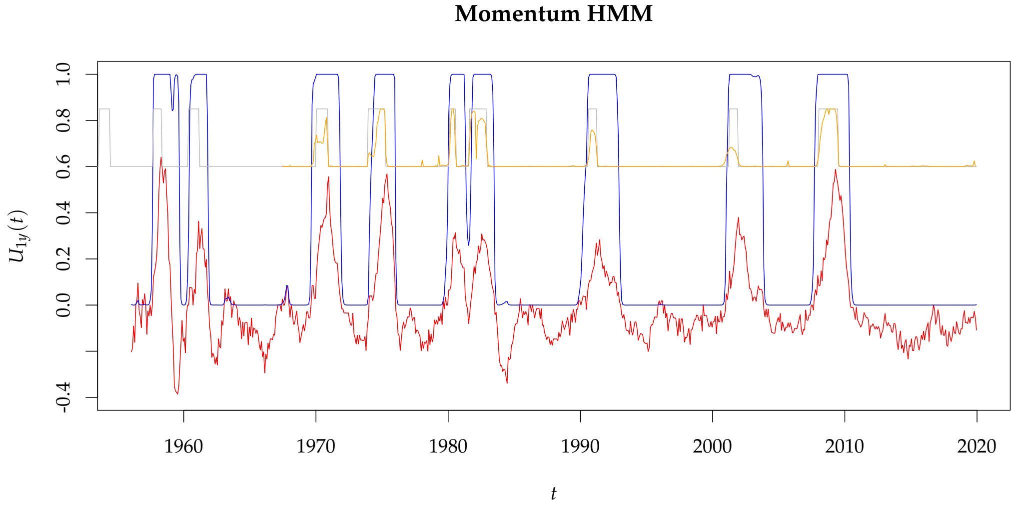

Figure 2: The HMM result of the Momentum model. The red line is $U_{1y}(t)$. The blue line is the probability of the crash state $P(t; S_U = 1)$. The gray line and orange line are the rescaled recession probabilities from NBER and Piger and Chauvet (2019) as references.

```
> rec$mom <- ldhmm.md2mle(rec$md_mom)
> rec.dtb$prob_mom <- rec$mom$prob
> rec$mom$data_stats

            mean         sd kurtosis   skewness length
[1,]  0.20370299 0.17692780 4.343996 -0.1362543    224
[2,] -0.08770653 0.06754179 3.385881 -0.4060775    544

> rec$mom$ld_stats

            mean         sd kurtosis
[1,]  0.19087509 0.17757382 3.975593
[2,] -0.08658013 0.06811048 3.314792
```

The specifics of two momentum states are shown in Table 1.

S_U: $U_{1y}(t)$ State	State Name	Mean (μ)	Volatility (SD)	Kurtosis	λ	σ
$S_U = 1$	Crash	0.19	0.18	4.0	1.41	0.20
$S_U = 2$	Normal	-0.087	0.068	3.3	1.14	0.090

Table 1: λ distribution parameters of two states in the Momentum model

The first state is the crash state that has large positive mean, large SD, and its kurtosis is nearly 4.0, which is not close to a normal distribution. The second state is the normal state with a small negative mean, small SD, and its kurtosis is close of that of a normal distribution.

We also observe that the stylized statistics from the data (`rec$mom$data_stats`) match reasonably well with the stylized statistics of the λ distribution states (`rec$mom$ld_stats`). The HMM result is plotted in Figure 2.

HMM States for the Acceleration Model

Next, we train the acceleration model. The following listing shows the statistics and parameters of the acceleration states from the `ldhmm` package:

```
> rec$acc <- ldhmm.md2mle(rec$md_acc)
> rec.dtb$prob_acc <- rec$acc$prob
> rec$acc$data_stats
```

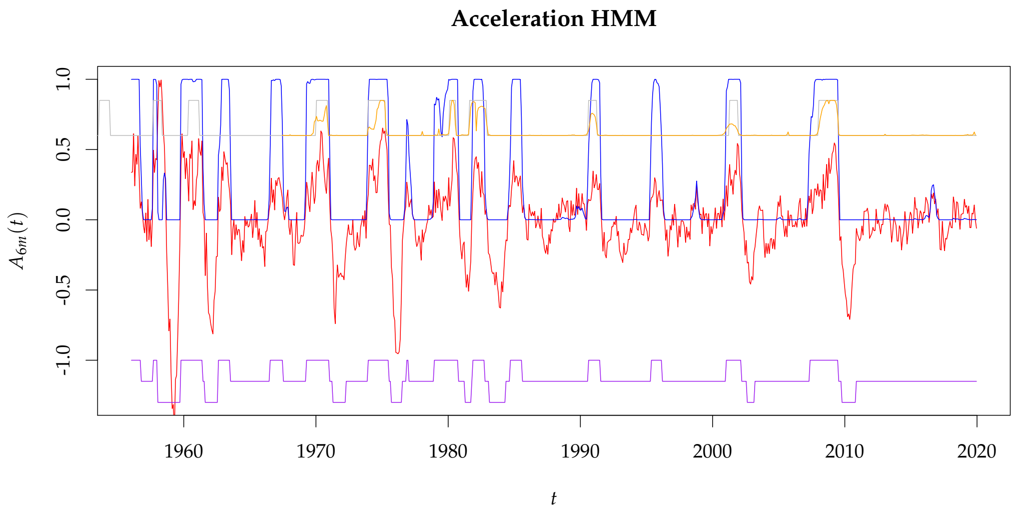

Figure 3: The HMM result of the Acceleration model. The red line is $A_{6m}(t)$. The blue line is the probability of the accelerating state $P(t; S_A = 1)$. The gray line and orange line are the rescaled recession probabilities from NBER and Piger and Chauvet (2019) as references. The purple line at the bottom illustrates the state transition.

```
            mean        sd kurtosis   skewness length
[1,]  0.29656765 0.1484197  2.45445  0.4120908    203
[2,] -0.03296496 0.1118862  2.48250 -0.1964831    469
[3,] -0.43560430 0.4461610  6.02684  1.1911775     96

> rec$acc$ld_stats

            mean        sd  kurtosis
[1,]  0.31411712 0.1535551  2.178124
[2,] -0.03384435 0.1120683  2.484404
[3,] -0.40497393 0.4514759 13.436038
```

The specifics of three acceleration states are shown in Table 2.

S_A: $A_{6m}(t)$ State	State Name	Mean (μ)	Volatility (SD)	Kurtosis	λ	σ
$S_A = 1$	Accelerating	0.31	0.15	2.2	0.48	0.27
$S_A = 2$	Middle	-0.035	0.11	2.5	0.70	0.18
$S_A = 3$	Decelerating	-0.40	0.45	13.6	3.15	0.11

Table 2: λ distribution parameters of three states in the Acceleration model

The accelerating state has large mean, with SD about half of the mean. This ensures the state capture the positive acceleration data. The middle state has a nearly zero mean. Both states have kurtosis slightly less than that of a normal distribution.

The decelerating state has large negative mean, with SD about the same magnitude as the mean. Thus this state captures the negative acceleration data. The most notable is that its kurtosis is very large. The data has it at 6.0, while its λ distribution has it at 13.0. This state is very far from a normal distribution.

The HMM result is plotted in Figure 3.

Calculating Recession Probability

The recession probability $P_{REC}(t)$ is the product of the HMM probability in the crash state $P(t; S_U = 1)$ from $U_{1y}(t)$ and the HMM probability in the accelerating state $P(t; S_A = 1)$ from $A_{6m}(t)$. That is,

$$P_{REC}(t) \equiv P(t; S_U = 1) \times P(t; S_A = 1)\,. \tag{4}$$

```
> rec.dtb$prob_recession <- rec.dtb$prob_mom * rec.dtb$prob_acc
```

Figure 4 shows the final result of $P_{REC}(t)$. As you can observe, it matches every recession in the past marked by NBER and Piger and Chauvet (2019) quite well.

We can calibrate the beginning and ending of each recession in HMM by the month when $P_{REC}(t)$ crosses 50%. Compared to official NBER data, the beginning and ending of each recession can differ by ± 4 months. However, the average beginning month is less than one month ahead of NBER. And the average ending month is less than one month later than NBER.

Discussion

We use a single macro-quantity UNRATE to calculate the recession probability. The model utilizes its first difference, momentum, and second difference, acceleration. The outcome is very close to the government benchmarks obtained from much more sophisticated procedures and models. Such result is satisfactory. It remains to be seen whether this model can flag the next recession as it occurs.

This model is a good example of unsupervised machine learning in finance. Financial markets, in particular factors, are full of regimes. The timeseries behave differently in different regimes. A model as such that can isolate the regimes using first and second differenecs can have potential applications elsewhere.

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Bibliography

Geum Il Bae, Woo Chang Kim, and John M. Mulvey (2014). *Dynamic asset allocation for varied financial markets under regime switching framework.* European Journal of Operational Research, Vol. 234, No. 2, pp. 450-458. [p1]

Lihn, Stephen H.-T. (2017). *Hidden Markov Model for Financial Time Series and Its Application to S&P 500 Index.* SSRN: 2979516. [p1]

Lihn, Stephen H.-T. (2019). *Business Cycles, Optimal Interest Rate, and Recession Forecast From Yield Curve, Unemployment, GDP, and Payrolls.* SSRN: 3422278. [p1]

NBER (2019). *NBER based Recession Indicators for the United States from the Period following the Peak through the Trough [USREC].* Retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/USREC. [p2]

John M. Mulvey and Han Liu (2016). *Identifying Economic Regimes: Reducing Downside Risks for University Endowments and Foundations.* The Journal of Portfolio Management, Fall 2016, Vol. 43, No. 1: pp. 100-108. [p1]

Piger, Jeremy Max and Marcelle Chauvet (2019). *Smoothed U.S. Recession Probabilities [RECPROUSM156N].* Retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/RECPROUSM156N. [p2, 5, 6, 7, 8]

R Core Team (2019). *R: A language and environment for statistical computing.* R Foundation for Statistical Computing, Vienna, Austria. URL http://www.R-project.org/. [p1]

U.S. Bureau of Labor Statistics (2019). *Civilian Unemployment Rate [UNRATE].* Retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/UNRATE. [p1]

Claudia Sahm (2019). *Direct Stimulus Payments to Individuals.* Board of Governors of the Federal Reserve System. A part of the book "Recession Ready: Fiscal Policies to Stabilize the American Economy", edited by Heather Boushey, Ryan Nunn, and Jay Shambaugh, May 2019. [p1]

Robert J. Shiller (2018). *Online Data for U.S. Stock Markets 1871-Present and CAPE Ratio.* http://www.econ.yale.edu/~shiller/data.htm

[p]

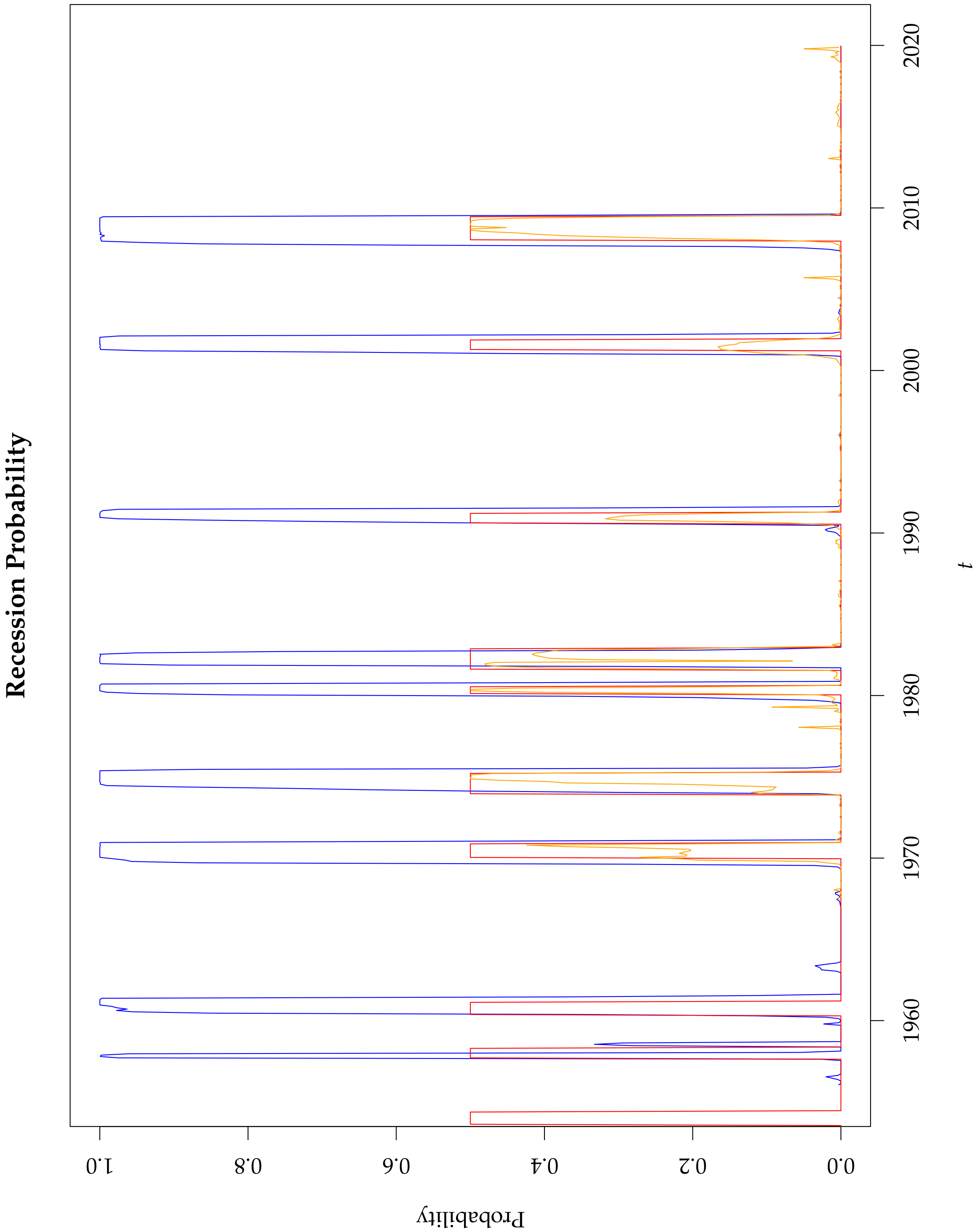

Figure 4: Recession Probability. The blue line is the recession probability $P_{REC}(t)$. The red line and orange line are the rescaled recession probability from NBER and Piger and Chauvet (2019) as references.